Limits Involving Infinity - Section 1.6

Defn: We say $\lim_{x \to c} f(x) = \infty$ if for every $M > 0$, $\exists \delta > 0$ s.t. for all $x \neq c$, if $|x - c| < \delta$, then $f(x) \geq M$

Example: $f(x) = \frac{1}{x^2}$ $\quad \lim_{x \to 0} \frac{1}{x^2} = \infty$

$\lim_{x \to \infty} \frac{1}{x^2} = 0$

Example: $\lim_{x \to 0^+} \frac{1}{x} = \infty$

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

Cannot say anything about $\lim_{x \to 0} \frac{1}{x}$!!

Finding Vertical Asymptotes

Ex) $f(x) = \frac{3x}{x^2 - 4}$ graph and plot asymptotes

Ex) $f(x) = \frac{(x-1)(x+1)}{(x-1)}$

Indeterminate Forms

$\lim_{x \to 0} \frac{\sin x}{x}$ $\quad \lim_{x \to 1} \frac{x^2 - 1}{x - 1}$ $\quad \Rightarrow \frac{0}{0}$

$\frac{0}{0}$ can give a limit that is ∞, 0, $x \in \mathbb{R}$ for any x.

List of Indeterminate Forms

$\infty - \infty$, $\infty \cdot 0$, $\frac{\infty}{\infty}$, $\frac{0}{0}$, 0^0

∞^0, 1^∞

Not indeterminate form

$\frac{1}{0}$, $\frac{\infty}{0}$

Horizontal Asymptotes

$$f(x) = \frac{x^2}{x^2+4} \qquad \lim_{x\to\infty} f(x) = 1$$

$$\lim_{x\to+\infty} f(x) = 1$$

Ex) $\lim_{x\to\infty} \frac{\sin(x)}{x} = 0$

Polynomials

$$f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0}$$

If: $n = m \Rightarrow \lim_{x\to-\infty}^{+\infty} f(x) = \frac{a_n}{b_m}$

$n < m \Rightarrow \lim_{x\to\pm\infty} f(x) = 0$

$n > m \Rightarrow \lim_{x\to\pm\infty} f(x)$ are infinite

$$\lim_{x\to\infty} \frac{x^2}{x^2+4} = \frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2} + \frac{4}{x^2}} = \frac{1}{1+\frac{4}{x^2}} = \frac{1}{1+0} = 1$$

Ex) $\lim_{x\to-\infty} \frac{x^2+2x-1}{x^3+1}$ Ex) $\lim_{x\to\infty} \frac{x^2+2x-1}{1-x-3x^2}$

Ex) $\lim_{x\to\infty} \frac{x^2-1}{3-x}$