Section 2.5 - Chain Rule

Chain Rule: Let $y = f(u)$ be differentiable function of u and let $u = g(x)$ be differentiable function of x.

Then $y = f(g(x))$ is differentiable wrt x and

$$y' = f'(g(x)) \cdot g'(x)$$

Ex) Find derivatives of

$y = (1-x)^2$ $\quad y = \ln(4x^3 - 2x^2)$ $\quad y = \cos(x^2)$

$y = \sin(2x)$ $\quad y = e^{-x^2}$

$y = \ln(3x^5 - \cos x + e^x)$

Ex) $y = x^5 \sin(2x^3)$

$y = \dfrac{5x^3}{e^{-x^2}}$ $\quad y = \tan^5(6x^3 - 7x)$

Derivatives of $y = a^x$

$$y = a^x \Rightarrow y = e^{x\ln(a)} \Rightarrow y' = e^{x\ln(a)}\ln(a)$$

$$y' = a^x \ln(a)$$

Rule: if $f(x) = a^x$, $a > 0$, $a \neq 1$, then $f'(x) = \ln(a)a^x$

Other ways to see chain rule

$$y'(x) = f'(g(x))\,g'(x)$$

$$\Rightarrow \frac{dy}{dx} = y'(u)\cdot u'(x) \qquad \text{if } y = f(u),\ u = g(x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$y'(x) = f'(g(h(x)))\,g'(h(x))\cdot h'(x) \qquad y = f(u),\ u = g(w),\ w = h(x)$$

$$\Rightarrow \frac{dy}{dx} = f'(u)\,u'(w)\,w'(x)$$

$$= \frac{dy}{du}\cdot\frac{du}{dw}\cdot\frac{dw}{dx}$$

Ex) $y = e^{e^{e^x}}$ $\qquad y = e^u,\ u = e^w,\ w = e^x$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dw}\cdot\frac{dw}{dx}$$

$$= e^u\cdot e^w\cdot e^x$$

$$= e^{e^w} e^{e^x} e^x$$

$$= e^{e^{e^x}} e^{e^x} e^x = \boxed{e^{e^{e^x}+e^x+x}}$$