Section 2.6 - Implicit Differentiation

Chain Rule: $\frac{d}{dx}(f(g(x)) = f'(g(x))g'(x)$

Let $y = g(x)$

$\Rightarrow \frac{d}{dx}(f(y)) = f'(y)\cdot y'$ or $\frac{d}{dx}(f(y)) = f'(y)\frac{dy}{dx}$

Find $\frac{dy}{dx} = y'$

Ex) $\sin(y) + y^3 = 6 - x^3$

Find tangent line at $(\sqrt[3]{6}, 0)$ Ans: $-\frac{3\sqrt[3]{36}}{1}$

Ex) $y^3 + x^2y^4 = 1 + 2x$

Ex) $\sin(x^2y^2) + y^3 = x + y$

Ex) $x^2 + y^2 = 1$ Tangent line at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $y' = \frac{-1/2}{\sqrt{3}/2} = -\frac{1}{\sqrt{3}}$

Ex) $x^{2/3} + y^{2/3} = 8$

Ex) $x^2 + y^2 = 1$ $\frac{d^2y}{dx^2} = y''$ $\Rightarrow y'' = \frac{d}{dx}(y')$

Ex) $y = x^x$

Ex) $y = (1+x)^{\frac{1}{x}} \Rightarrow$ $\ln(y) = \frac{1}{x}\ln(1+x) = x^{-1}\ln(1+x)$

$\frac{1}{y}\frac{dy}{dx} = \left(-x^{-2}\ln(1+x) + x^{-1}\frac{1}{x+1}\right)$