LAST NAME: FIRST NAME: KEY

Math 008B - Spring 2016

Quiz 3: Wednesday April 27, 2016

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1. (5 points) For what value of c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx + 4 & \text{if } x \leq 2 \\ cx^2 + 2 & \text{if } x > 2 \end{cases}$$

$\lim_{x \to 2} cx+4 = 2c+4$

$\lim_{x \to 2} cx^2+2 = 4c+2$

$\Rightarrow 4c+2 = 2c+4$

$2c = 2$

$c = 1$

Limit exists when $c = 1$

$\Rightarrow f(x) = \begin{cases} x+4 & \text{if } x \leq 2 \\ x^2+2 & \text{if } x > 2 \end{cases}$

$f(2) = 2+4 = 6 \Rightarrow f(c)$ is defined

$\lim_{x \to 2} f(x) = 6$ from above so $\lim_{x \to 2} f(x) = f(2)$ ✓

Please show all work.

2. (5 points) Let $f(x) = x^2 - 1$.

a) Use the limit process (difference quotient) to find the slope of the line tangent to the graph of $f(x)$ at $x = 1$.

b) Find the equation of the line tangent to the graph of $f(x)$ at $x = 1$.

a) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{(x+h)^2-1-(x^2-1)}{h} = \dfrac{x^2+2xh+h^2-1-x^2+1}{h}$

$= \dfrac{2xh+h^2}{h} = 2x+h$

$\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} 2x+h = 2x$

Slope is $f'(1) = 2$

b) $y - y_1 = m(x - x_1)$ $\quad m = 2$ from part (a)

$x_1 = 1 \Rightarrow f(x_1) = x_1^2 - 1$

$f(1) = 1^2 - 1$

$y_1 = 0$

$y - 0 = 2(x-1)$

$\Rightarrow \boxed{y = 2x - 2}$

Please show all work.