Name: _____
Section: _____

Math 46: Quiz 4
February 19, 2015

Please show all work and solve the following problems.

1. A body at a tempurature of 0 degrees F is placed in a room whose temperature is kept at 100 degrees F. If after 10 minutes the temperature of the body is 25 degrees F, find

   (a) the time required for the body to reach a temperature of 50 degrees F, and

   (b) the temperature of the body after 20 minutes.

   ***Solution*:** (a) The ODE that we are given is

   $$\frac{dT}{dt} + kT = kT_m$$

   This is a first order linear ODE, which has the solution

   $$T(t) = T_m + (T_0 - T_m)e^{-kt}$$

   where T_0 is the intial temperature and T_m is the surrounding temperature. So from the story, we know that $T_0 = 0$ and $T_m = 100$. Our solution so far is

   $$T(t) = 100 - 100e^{-kt}$$

   Now using the last piece of information, we solve for k, which is $T(10) = 25$. So then we have

   $$\begin{aligned} 25 &= 100 - 100e^{-10k} \\ -75 &= -100e^{-10k} \\ .75 &= e^{-10k} \\ -\frac{\ln(.75)}{10} &= k \end{aligned}$$

   So the equation is

   $$T(t) = 100 - 100e^{\frac{\ln(.75)}{10}t}$$

Now we use the info from the question to find the time:

$$\begin{aligned} 50 &= 100 - 100e^{\frac{\ln(.75)}{10}t} \\ -50 &= -100e^{\frac{\ln(.75)}{10}t} \\ \frac{1}{2} &= e^{\frac{\ln(.75)}{10}t} \\ \ln\left(\frac{1}{2}\right) &= \frac{\ln(.75)}{10}t \\ t &= \frac{10}{\ln(.75)}\ln\left(\frac{1}{2}\right) \end{aligned}$$

(b) For part b, all we do is plug 20 into the function:

$$\begin{aligned} T(t) &= 100 - 100e^{\frac{\ln(.75)}{10}20} \\ &= 100 - 100e^{2\ln(.75)} \end{aligned}$$

□