MATH 046

Extra Problem

Solve $y\,dx - x\,dy = 0$ $(*)$

Using the integrating factor $\frac{1}{y^2}$, we get

$$\frac{1}{y^2}\left(y\,dx - x\,dy = 0\right)$$

$$\Rightarrow \frac{1}{y}\,dx - \frac{x}{y^2}\,dy = 0 \Rightarrow M(x,y)\,dx + N(x,y)\,dy = 0$$

$$\Rightarrow \text{So} \quad M(x,y) = \frac{1}{y} \quad \text{and} \quad N(x,y) = \frac{-x}{y^2}$$

$$\Rightarrow \frac{\partial M}{\partial y} = -\frac{1}{y^2} \quad \text{and} \quad \frac{\partial N}{\partial x} = -\frac{1}{y^2}$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

So then we have $\frac{\partial g}{\partial x} = M = \frac{1}{y}$ $\quad \frac{\partial g}{\partial y} = N = -\frac{x}{y^2}$ (☆)

First integrate $\frac{\partial g}{\partial x}$ to find g by Fund. Thm of Calculus

$$\Rightarrow \int \frac{\partial g}{\partial x}\,dx = \int \frac{1}{y}\,dx$$

$$g(x,y) = \frac{x}{y} + h(y)$$

Now we use (☆) on the above formula

$$\frac{\partial g(x,y)}{\partial y} = -\frac{x}{y^2} + h'(y) = N = \frac{-x}{y^2}$$

$$\Rightarrow h'(y) = 0 \Rightarrow h(y) = C \text{, a constant}$$

$$\Rightarrow g(x,y) = \frac{x}{y} + C \quad \text{is the solution to } (*)$$

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