Homework #1 - 146C

10.1) 2, 3, 6, 7, 12, 14, 16, 19, 23

10.2) 10, 14, 16, 27

10.1) 16 = 3.5 pts

10.2) 14 = 3.5

neatness 1 pt

~~correctness~~ 2 pts

completeness 0 / <

.5	~
1	~ 5-7
1.5	~ 8-10
2	100%, -1

2) $y'' + 2y = 0 \quad y'(0) = 1, \; y'(\pi) = 0$

$y(x) = c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x$

$y'(x) = -c_1\sqrt{2}\sin\sqrt{2}x + c_2\sqrt{2}\cos\sqrt{2}x$

$1 = c_2\sqrt{2} \Rightarrow y'(x) = -c_1\sqrt{2}\sin\sqrt{2}x + \cos\sqrt{2}x$

$0 = -c_1\sqrt{2}\sin\sqrt{2}\pi + \cos\sqrt{2}\pi$

$c_1 = \frac{1}{\sqrt{2}}\cot\sqrt{2}\pi$

$\boxed{\text{Unique}}$

3) $y'' + y = 0 \quad y(0) = 0, \; y(L) = 0$

$y(x) = c_1\cos x + c_2 \sin x$

$0 = c_1 \Rightarrow y(x) = c_2 \sin x$

$0 = c_2 \sin(L)$

$\Rightarrow c_2 = 0$

$\boxed{\text{only trivial sol'n s}}$

6) $y'' + 2y = x \quad y(0) = 0, \; y(\pi) = 0$

$y(x) = \frac{x}{2} + c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x$

$0 = c_1 \Rightarrow y(x) = \frac{x}{2} + c_2\sin\sqrt{2}x$

$0 = \frac{\pi}{2} + c_2\sin\sqrt{2}\pi$

$\boxed{\text{Unique}}$

7) $y'' + 4y = \cos x \quad y'(0) = 0, \; y'(\pi) = 0$

$y(x) = c_1\cos(2x) + c_2\sin(2x) + \frac{1}{12}\left(4\cos^3 x \cos 2x + 3\sin x \sin 2x + \sin 2x \sin 3x\right)$

$y' = -c_1 2\sin(2x) + 2c_2\cos(2x)$

$= \Big|_0^x \qquad = \Big|_0^x$

ila

23) $y'' + \lambda y = 0$

$y(0) = 0\ ,\ y(\pi) = 0$

a) $y = k_1 e^{i\mu x} + k_2 e^{-i\mu x}$ $\qquad \lambda = \mu^2$

$\Rightarrow \quad k_1 + k_2 = 0$

$k_1 e^{i\mu\pi} + k_2 e^{-i\mu\pi} = 0$

System has nontrivial soln iff coeff matrix is singular

$\Rightarrow \det = 0$

$$\begin{bmatrix} 1 & 1 \\ e^{i\mu\pi} & e^{-i\mu\pi} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \det A = e^{-i\mu\pi} - e^{i\mu\pi} = 0$

b) Let $\mu = \nu + i\sigma$

$\Rightarrow i\mu\pi = i\nu\pi - \sigma\pi$

$\Rightarrow e^{\sigma\pi} e^{-i\nu\pi} - e^{-\sigma\pi} e^{i\nu\pi} = 0$

Euler $\Rightarrow e^{\sigma\pi}(\cos\nu - i\sin\nu) - e^{-\sigma\pi}(\cos\nu + i\sin\nu) = 0$

Re, Im $\Rightarrow (e^{\sigma\pi} - e^{-\sigma\pi})\cos\nu\pi = 0$ ①

$(e^{\sigma\pi} + e^{-\sigma\pi})\sin\nu\pi = 0$ ②

c) By ②, $\nu \in \mathbb{Z}$, $\cos\nu\pi \neq 0$

$\Rightarrow e^{\sigma\pi} - e^{-\sigma\pi} = 0 \Rightarrow e^{\sigma\pi} = e^{-\sigma\pi}$

$\Rightarrow e^{2\sigma\pi} = 1 \Rightarrow \sigma = 0$

$\mu = \nu = n$

27) $S = x+T \Rightarrow x = S-T$

a) $\int_0^T g(x)\,dx = \int_0^a g(x)\,dx + \int_a^T g(x)\,dx$

$= \int_T^{a+T} g(s-T) + \int_a^T g(x)\,dx$

$= \int_T^{a+T} g(s) + \int_a^T g(x)\,dx$

$s = x$ $\quad = \int_T^{a+T} g(x) + \int_a^T g(x)\,dx$

$= \int_a^{a+T} g(x)$

b) a arbitrary, $T > 0 \Rightarrow \exists n$ s.t. $nT \leq a < (n+1)T$

$\Rightarrow 0 \leq a - nT < T$

Use part (a), cov $\cdot\, y = x + nT$

c) a arb $\Rightarrow \int_0^T g(x)\,dx = \int_a^{a+T} g(x)\,dx$

b arb $\Rightarrow \int_0^T g(x)\,dx = \int_b^{b+T} g(x)\,dx$

$\Rightarrow$ result.

10.2

16) $f(x) = \begin{cases} x+1 & -1 \le x < 0 \\ 1-x & 0 \le x < 1 \end{cases}$ $\quad f(x+2) = f(x)$

$a_0 = \int_{-1}^{0}$