Lie n-algebras, supersymmetry and division algebras

John Huerta

There is a relationship between normed division algebras and certain supersymmetric theories of physics which lies at the heart of the following pattern:

- The only normed division algebras are $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$ and $\mathbb{O}$. They have dimensions $k = 1$, 2, 4 and 8.
- The classical superstring makes sense only in spacetimes of dimension $k + 2 = 3$, 4, 6 and 10.
- The classical super-2-brane makes sense only in spacetimes of dimension $k + 3 = 4$, 5, 7 and 11.

I will sketch how to use the normed division algebras to prove the spinor identities necessary for the existence of the classical superstring and 2-brane theories. Then I will describe how *exactly the same mathematics* implies the existence of certain higher structures, namely:

- In the superstring dimensions $k + 2 = 3$, 4, 6 and 10, we can use the normed division algebras to construct a Lie 2-superalgebra $\mathfrak{superstring}$ which extends the Poincaré Lie superalgebra in these dimensions.
- In the super-2-brane dimensions $k + 3 = 4$, 5, 7 and 11, we can use the normed division algebras to construct a Lie 3-superalgebra 2-$\mathfrak{brane}$ which extends the Poincaré Lie superalgebra in these dimensions.

References

[1] J. Baez and J. Huerta, Division algebras and supersymmetry I, to appear in *Proceedings of the NSF/CBMS Conference on Topology, C* Algebras, and String Theory*. Available as `arXiv:0909.0551`.

[2] J. Baez and J. Huerta, Division algebras and supersymmetry II. Available as `arXiv:1003.3436`.