Abelian Sheaves and Picard Stacks

In SGA4 Exposé XVIII, Deligne studies the relation between Picard stacks and length 2 complexes of abelian sheaves, as well as the relation between the morphisms of such objects. He proves that the functor

$$\mathrm{D}^{[-1,0]}(\mathsf{S}) \longrightarrow \mathrm{P\textsc{ic}}^{\flat}(\mathsf{S})$$

is an equivalence. $\mathrm{D}^{[-1,0]}(\mathsf{S})$ is the subcategory of the derived category of category of complexes of abelian sheaves $A^{\bullet}$ over a site S with $H^{-i}(A^{\bullet}) \neq 0$ only for $i = 0, 1$ and $\mathrm{P\textsc{ic}}^{\flat}(\mathsf{S})$ is the category of Picard stacks over S with 1-morphisms isomorphism classes of additive functors.

The purpose of this talk is to generalize the above result to Picard 2-stacks. We give a definition of Picard 2-stack and define their 3-category $2\mathrm{P\textsc{ic}}(\mathsf{S})$. We also introduce a tricategory $\mathrm{T}^{[-2,0]}(\mathsf{S})$ of length 3 complexes of abelian sheaves. Then we construct a trihomomorphism

$$\mathrm{T}^{[-2,0]}(\mathsf{S}) \longrightarrow 2\mathrm{P\textsc{ic}}(\mathsf{S}),$$

which we prove to be a triequivalence. From this triequivalence, we deduce a generalization of Deligne's analogous result about Picard stacks.