Base Conversion Guide

UCR · Math 135A

Decimal (Base 10)	Binary (Base 2)	Octal (Base 8)	Hexidecimal (Base 16)
0	0000	00	00
1	0001	01	01
2	0010	02	02
3	0011	03	03
4	0100	04	04
5	0101	05	05
6	0110	06	06
7	0111	07	07
8	1000	10	08
9	1001	11	09
10	1010	12	0A
11	1011	13	0B
12	1100	14	0C
13	1101	15	0D
14	1110	16	0E
15	1111	17	0F

1. **Convert from base β to base 10.**
   Integer Part:

$$\begin{aligned}(a_n a_{n-1} \cdots a_1 a_0)_\beta &= a_n * \beta^n + a_{n-1} * \beta^{n-1} + ... + a_1 * \beta^1 + a_0 * \beta^0 \\ &= (x)_{10}\end{aligned}$$

   Fraction Part:

$$\begin{aligned}(0.b_1 b_2 b_3 \cdots)_\beta &= b_1 * \beta^{-1} + b_2 * \beta^{-2} + b_3 * \beta^{-3} + ... \\ &= (x)_{10}\end{aligned}$$

   Example: Convert 21.112 in base 3 to base 10

$$\begin{aligned}(21.112)_3 &= 2 * 3^1 + 1 * 3^0 + 1 * 3^{-1} + 1 * 3^{-2} + 2 * 3^{-3} \\ &= 2 * 3 + 1 * 1 + 1 * .333 + 1 * .111 + 2 * .037 \\ &= 6 + 1 + .333 + .111 + .074 \\ &= \boxed{(7.518...)_{10}}\end{aligned} \tag{1}$$

2. **Convert from base 10 to base β.**
Integer Part:

   (a) Divide the number by β and record the remainder.
   (b) Divide the resulting quotient by β and record the remainder.
   (c) Repeat
   (d) The number in base β is the remainders written in backwards order.

   Fraction Part:

   (a) Multiply the number by β and record the integer.
   (b) Multiply the resulting number (ignoring the integer) by β and record the integer.
   (c) Repeat
   (d) The number in base β is the integers written in forwards order.

   Example: Convert 15.4375 in decimal (base 10) to binary (base 2)

   - First convert the integer part:
   $$15 \div 2 = 7\ R\mathbf{1}$$
   $$7 \div 2 = 3\ R\mathbf{1}$$
   $$3 \div 2 = 1\ R\mathbf{1}$$
   $$1 \div 2 = 0\ R\mathbf{1}$$
   Hence $(15)_{10} = (1111)_2$.
   - Second convert the fraction part:
   $$0.4375 * 2 = \mathbf{0}.8750$$
   $$0.875 * 2 = \mathbf{1}.750$$
   $$0.75 * 2 = \mathbf{1}.50$$
   $$0.5 * 2 = \mathbf{1}.0$$
   Hence $(0.4375)_{10} = (0.0111)_2$
   - Final Answer: $(15.4375)_{10} = \boxed{(1111.0111)_2}$

3. **Convert from base α to base β.**

   - First convert from base α to base 10.
   - Second convert from base 10 to base β.

4. **Convert from decimal to single precision machine representation.**
Steps:

(a) Identify the sign (length 1):
- 0 for +
- 1 for -

(b) Identify the mantissa (length 23):
- First, convert the decimal number (without the sign) to binary. It is recommended to convert from base 10 to 8 to 2.
- Second, move the decimal forwards or backwards so that it is written in the the form $a.b_1 b_2 b_3 \cdots \times 2^n$ where n is the number of spaces you moved the decimal (can be positive or negative depending on the direction the decimal was moved).
- The mantissa is the number $b_1 b_2 b_3 \cdots$
- Add enough zeros to the mantissa so that it is 23 digits long.

(c) Identify the exponent (length 8):
- Solve $c - 127 = n$ for c. You know n from calculation of the mantissa.
- Note that c is in base 10. Convert it to binary. Again it is recommended to convert from base 10 to 8 to 2.
- If there is a leading 0, eliminate is so you have a number of 8 digits in length.

(d) Put it all together:
- Put the value for the sign in the first slot.
- Next write down the 8 digits of the number for the exponent step.
- Lastly write down the mantissa with the extra zeros so the number has a total of 32 digits.

(e) Convert to hexidecimal:
- Divide the 32 digit number into 8 numbers each of length 4.
- Convert each 4 digit number from binary to hexidecimal. This is your final answer.

Example: Convert -52.234375 to single precision machine representation.

(a) Identify the sign: The number is negative, which implies 1.

(b) Identify the mantissa:
- We will convert 52.234375 to binary:
  First convert the integer part:
  $$52 \div 8 = 6 \ R\mathbf{4}$$
  $$6 \div 8 = 0 \ R\mathbf{6}$$
  So $(52)_{10} = (64)_8 = (110\ 100)_2$
  Second convert the fraction part:
  $$0.234375 * 8 = \mathbf{1}.875000$$
  $$0.875 * 8 = \mathbf{7}.000$$
  So $(0.234375)_{10} = (0.17)_8 = (0.001\ 111)_2$.

- This implies $(52.234375)_{10} = (110100.001111)_2 = (1.10100001111)_2 \times 2^5$
- The mantissa with added zeros is then 10100001111000000000000.

(c) Identify the exponent:

- First we solve for c: $c - 127 = 5$ implies $c = (132)_{10}$
- Second we convert 132 from base 10 to binary.

$$132 \div 8 = 16 \ R\mathbf{4}$$

$$16 \div 8 = 2 \ R\mathbf{0}$$

$$2 \div 8 = 0 \ R\mathbf{2}$$

So $(132)_{10} = (204)_8 = (010\ 000\ 100)_2 = (10\ 000\ 100)_2$

- The exponent is 10000100.

(d) Put it all together: First list the number from step (a), then step (c), then step (b):

$$(11000010010100001111000000000000)_2$$

(e) Convert to hexidecimal:

- First split into numbers of length 4:

$$(1100\ 0010\ 0101\ 0000\ 1111\ 0000\ 0000\ 0000)_2$$

- Second convert each 4 digit number to hexidecimal to get the answer:

$$\boxed{(C250F000)_{16}}$$

5. **Convert from single precision machine representation to decimal.**
Steps:

(a) Convert the machine representation in hexidecimal to binary.

(b) Identify the sign, exponent, mantissa
- sign: first digit
- exponent: next 8 digits
- mantissa: last 23 digits (can ignore trailing zeros)

(c) Convert the sign
- 0 represents +
- 1 represents -

(d) Convert the exponent
- Convert the exponent number from binary to decimal, call the number in base 10 c.
- Calculate $c - 127$, call this number n.

(e) Convert the mantissa
- Suppose your mantissa is $b_1 b_2 \cdots b_m$, then write

$$\pm 1.b_1 b_2 \cdots b_m \times 2^n$$

where the sign is determined by step (c).
- Convert this number to standard form (i.e. not scientific notation).
- Convert this number to base 8, then to base 10, i.e. decimal. This is your final answer.

Example: Convert the single precision machine representation $(BA390000)_{16}$ to decimal.

(a) Convert the machine representation to binary:

$$(BA390000)_{16} = (1011\ 1010\ 0011\ 1001\ 0000\ 0000\ 0000\ 0000)_2$$

(b) Identify the sign, exponent, mantissa:
- sign: 1
- exponent: 01110100
- mantissa (without trailing zeros): 0111001

(c) Convert the sign: 1 represents - (negative)

(d) Convert the exponent:
- Convert from base 2 to base 10

$$\begin{aligned}(01\ 110\ 100)_2 &= (164)_8 \\ &= 1 * 8^2 + 6 * 8^1 + 4 * 8^0 \\ &= 64 + 48 + 4 \\ &= (116)_{10}\end{aligned}$$

- Let $c = 116$, the $c - 127 = 116 - 127 = -11$

(e) Convert the mantissa:

- The sign is negative, the mantissa is 0111001, and $n = -11$, so

$$-(1.0111001)_2 \times 2^{-11} = -(0.000\ 000\ 000\ 010\ 111\ 001)_2$$

- Convert this number from base 2 to decimal.

$$\begin{aligned} -(0.000\ 000\ 000\ 010\ 111\ 001)_2 &= -(0.000271)_8 \\ &= -(2 * 8^{-4} + 7 * 8^{-5} + 1 * 8^{-6}) \\ &= \boxed{-7.0571899 \times 10^{-4}} \end{aligned}$$