MathConnections 2018

May 19th, 2018
University of California, Riverside

Geometry & Differential Equations

UC RIVERSIDE | UNIVERSITY OF CALIFORNIA | Department of Mathematics

Program

Welcome	
8:00 – 9:00 AM	Registration | Surge 2nd Floor Entry Coffee & Tea | Surge 284

Morning Session

Session A | Surge 284	
9:00 – 9:30 AM	**Michael McNulty** | UC Riverside *"Development of singularities of the Skyrme model"*
9:30 – 10:00 AM	**Xavier Ramos Olivé** | UC Riverside *"Gradient estimates and integral Ricci Curvature"*
10:00 – 10:30 AM	**Alex Mramor** | UC Irvine *"On the topological rigidity of self shrinkers in $\mathbb{R}^3$"*
10:30 – 11:00 AM	**Matt Gibson** | UC Irvine *"Investigating a sympletctic cohomology and its relationship to the mapping class group"*

Session B | Surge 268	
9:00 – 9:30 AM	**Christian Williams** | UC Riverside *"Categorical computation"*
9:30 – 10:00 AM	**Christina Knox** | UC Riverside *"Determining both the source of a wave and its speed in a medium from boundary measurements"*
10:00 – 10:30 AM	**Tommy Trakoolthai** | Cal Poly Pomona *"A multivariate log-normal moment closure technique for the stochastic predator-prey model"*
10:30 – 11:00 AM	**Wei-Xiang Feng** | UC Riverside *"On the existence of Buchdahl's stability bound in EiBI Gravity"*

Morning Keynote Presentation | Surge 284	
11:00 – 12:00 PM	**Dr. Richard Schoen** | UC Irvine *"From Newtonian gravity to general relativity"*

12:00 – 1:00 PM	Lunch | Surge 268

Afternoon Session

Afternoon Keynote Presentation | Surge 284	
1:00 – 2:00 PM	**Dr. Po-Ning Chen** | UC Riverside *"Energy in general relativity"*

Session A | Surge 284	
2:00 – 2:30 PM	**Lawrence Mouillé** | UC Riverside *"Symmetry-rank and positive curvature"*
2:30 – 3:00 PM	**Yihan Li** | UC Santa Barbara *"Asmptotic spectral flow with heat equation method"*
3:00 – 3:30 PM	**Yousef Chahine** | UC Santa Barbara *"Volume inequalities with intermediate Ricci curvature"*
3:30 – 4:00 PM	**Casey Blacker** | UC Santa Barbara *"Vector-valued symplectic geometry and the moduli space of flat connections"*
4:00 – 4:30 PM	**Jiawei Zhou** | UC Irvine *"Massey products on sphere bundles over formal manifolds"*
4:30 – 5:00 PM	**Pilar Orellana** | UC Riverside *"Kähler-Einstein metrics on compact cohomogeneity on Fano manifolds via effective approximations"*

Session B | Surge 268	
2:00 – 2:30 PM	**Tim McEldowney** | UC Riverside *"Hilbert and Jacobson module equivalence"*
2:30 – 3:00 PM	**Daniel Cicala** | UC Riverside *"A low key introduction to synthetic differential geometry"*
3:00 – 3:30 PM	**Jade Master** | UC Riverside *"Chemical reachability in open Petri nets"*
3:30 – 4:00 PM	**Ryan Ta** | UC Riverside *"Existence of a global solution to the relativistic Vlasov-Poisson equation in dimension three"*
4:00 – 4:30 PM	**Meijke Balay-Mickelson** | UC Los Angeles *"The Krzyż conjecture and related coefficient problems"*
4:30 – 5:00 PM	**Chon In Luk** | Cal Poly Pomona *"Determining the probability of all lattice sample paths of an alternating birth-death chain traveling from state i to j within a strip"*

Abstracts

Keynote Speakers | Surge 284

11:00 AM - **Dr. Richard Schoen** | UC Irvine

"From Newtonian gravity to general relativity"

This will be a talk intended for graduate students with interest in geometric analysis. We will describe the basics of the Einstein equations, and discuss the Schwarzschild solution, and explain how it leads to a general notion of total gravitational mass (ADM mass) for certain general classes of solutions. We will assume basic knowledge of Riemannian geometry.

1:00 PM - **Dr. Po-Ning Chen** | UC Riverside

"Energy in general relativity"

While the concept of energy is fundamental in any physical theory, it is one of the outstanding problem in the theory of general relativity due to the equivalence principle. In this talk, we will discuss how to use geometric analysis to help us find solution to this outstanding problem in general relativity.

Morning Session A | Surge 284

9:00 AM - **Michael McNulty** | UC Riverside

"Development of singularities of the Skyrme model"

The Skyrme model is a geometric field theory and a quasilinear modification of the Nonlinear Sigma Model (Wave Maps). In this talk we study the development of singularities for the equivariant Skyrme Model, in the strong-field limit, where the restoration of scale invariance allows us to look for self-similar blow-up behavior. After introducing the Skyrme Model and reviewing what's known about formation of singularities in equivariant Wave Maps, we prove the existence of smooth self-similar solutions to the $5+1$-dimensional Skyrme Model in the strong-field limit, and use that to conclude that the solution to the corresponding Cauchy problem blows up in finite time, starting from a particular class of everywhere smooth initial data.

9:30 AM - **Xavier Ramos Olivé** | UC Riverside

"Gradient estimates and integral Ricci curvature"

Estimates on the gradient of solutions to elliptic and parabolic equations in $\mathbb{R}^n$ are fundamental results with lots of applications: from proving Harnack's inequalities, to finding lower bounds on eigenvalues for different boundary value problems. With a lower bound assumption on the Ricci curvature, these results were generalized to manifolds by P. Li, S.T. Yau, and others, discovering several applications of these estimates to geometry.

Recently, Q.S. Zhang and M. Zhu weakened the pointwise assumption of a lower bound on the Ricci curvature to a bound on its negative part in an Lp sense: an integral Ricci curvature bound. We will discuss how integral Ricci curvature bounds can be used to derive some of these estimates,

the key results needed to derive them on a manifold, and some possible applications and open directions.

9:00 AM - **Alex Mramor**, Shengwen Wang | UC Irvine

"On the topological rigidity of self shrinkers in $\mathbb{R}^3$"

Self shrinkers are singularity models for the mean curvature flow and thus are of central importance in the field. They happen to be minimal surfaces in an appropriate conformal change of the Euclidean metric. In this talk we will explain how, in analogy to the positive Ricci curvature setting, they enjoy the Frankel property and how then to see that any compact self shrinker in $\mathbb{R}^3$ is a Heegard splitting in an appropriate sense. Next we will explain how, using a deep theorem of Waldhausen, any self shrinker is “unknotted” in that any closed self shrinker of $\mathbb{R}^3$ of genus g is ambiently isotopic to the standard genus g surface in $\mathbb{R}^3$. If time permits we will conclude with a number of (to the speakers knowledge) open questions.

10:30 AM - **Matt Gibson** | UC Irvine

"Investigating a symplectic cohomology and its relationship to the mapping class group"

Tsai, Tseng, and Yau have developed a novel cohomology on symplectic manifolds. In this talk I will briefly introduce its construction along with recent developments for symplectic manifolds with boundary. As an application I present two homeomorphic symplectic 4-folds, coming from a torus with four boundary components, which are distinguished by the dimension of the cohomology groups.

Afternoon Session A | Surge 284

2:00 PM - **Lawrence Mouillé** | UC Riverside

"Symmetry-rank and positive curvature"

The Grove symmetry program has been an immense source of synergy in the field of Riemannian geometry. The goal of the program is to ``classify" positively curved manifolds with ``large" isometry groups. One of the first results in this direction is the Grove-Searle maximal symmetry-rank theorem which states that if a closed n-manifold has positive sectional curvature, then its symmetry-rank (the rank of the its isometry group) is $\leq \lfloor (n+1)/2 \rfloor$, and in the case of equality, the manifold is diffeomorphic to a quotient of a sphere. In this talk, I will show that a generalized bound on the symmetry-rank holds for manifolds with positive intermediate Ricci curvature, a condition on the curvature tensor that interpolates between having positive sectional curvature and having positive Ricci curvature. I will also present current work-in-progress toward understanding the consequences of this result.

2:30 PM - **Yihan Li** | UC Santa Barbara

"Asymptotic spectral flow with heat equation method"

Asymptotic spectral flow is first studied by Taubes in his proof of Weinstein conjecture in dimension 3 and has been later generalized for Dirac operators over all odd dimensional Spin manifolds. In my project, we used Getzler rescaling and a uniform estimate of heat kernels for a family of

rescaled operators introduced by Dai-Liu-Ma to find a new approach and some improvement of the estimate.

3:00 PM - **Yousef Chahine** | UC Santa Barbara

"Volume inequalities with intermediate Ricci curvature"

A classical inequality of Heintze and Karcher in Riemannian geometry gives an upper bound for the volume of a geodesic tube around a submanifold in terms of its mean curvature and lower bounds on the sectional curvature of the ambient space. For a submanifold of arbitrary codimension, one can weaken the sectional curvature bound to a k-Ricci curvature bound. After reviewing the comparison theory for pointwise k-Ricci curvature bounds (based on the comparison theory for a Riccati differential equation) I will give a volume estimate for geodesic tubes around minimal submanifolds in spaces where the negative part of the k-Ricci curvature is small with respect to certain integral norms.

3:30 PM - **Casey Blacker** | UC Santa Barbara

"Vector-valued symplectic geometry and the moduli space of flat connections"

Let G be a compact semisimple Lie group, Σ a compact oriented surface, and P a G-principal bundle on Σ. It is well-known that the space $\mathcal{A}(P)$ of connections on P is equipped with a natural symplectic structure, with respect to which the moduli space $\mathcal{M}(P)$ of flat connections is the symplectic reduction. Technical considerations require that the base space Σ be a surface. However, with respect to a certain generalized symplectic structure which takes values in a suitable vector space V, the moduli space of flat connections over an arbitrary manifold M of dimension at least 3 can again be exhibited as the symplectic reduction of $\mathcal{A}(P)$ by the gauge group.

4:00 PM - **Jiawei Zhou** | UC Irvine

"Massey products on sphere bundles over formal manifolds"

Given a formal differential graded algebra (dga), we can do an elementary extension by adding one odd-degree generator. The new dga may not be formal, but we give a construction of an A-infinity minimal model with only m_2 and m_3 non-trivial. As a consequence, the n-point Massey products vanish for $n > 3$ and in particular they must vanish on an odd-dimensional sphere bundle over a formal manifold. As an application we show the higher ($n > 3$) Massey products vanish on the filtered cohomology, constructed by Tsai, Tseng and Yau, of a formal symplectic manifold.

4:30 PM - **Pilar Orellana** | UC Riverside

"Kähler-Einstein metrics on compact cohomogeneity on Fano manifolds via effective approximations"

Kähler-Einstein metrics emerge when a complex, topological manifold, under additional conditions, admits a metric that is both Einstein and Kähler. They are beautiful objects which arise naturally in many facets of mathematics--and moreover, are of great importance in the study of string theory. We want to determine under what conditions a compact Fano manifold of Type I cohomogeneity one admits Kähler-Einstein metrics; for which is done by verifying the classes of the manifolds being Fano manifolds and their stability; however, by using the standard methods

currently available to us, this proves to be quite a cumbersome task which yields very limited results. In order to overcome this obstacle, we have developed new specialized methods which are effective at retrieving large-scale information of classes of these compact Fano manifolds and their corresponding Kähler-Einstein properties.

Morning Session B | Surge 268

10:00 AM - **Christian Williams** | UC Riverside

"Categorical computation"

Logic is the science of reasoning, Type Theory is the science of formal systems, and Category Theory is the science of interrelation. The three are deeply intertwined in the foundations of abstract thought. Modern theoretical computer science is inspired and guided by these unifying principles, and the developing ideas can reform the systems on which today's society is built. RChain is a distributed network, which dramatically reimagines the global virtual world. We discuss a particular innovation of this project, which exemplifies the deep harmony of theory and practice characteristic of this ideological movement. ``Logic as a distributive law" is a categorical algorithm which generates sound type systems for formal languages, and also functions as a powerful generalized query language. No knowledge is assumed; the only goal is to spread the word about some great ideas.

9:30 AM - **Christina Knox** | UC Riverside

"Determining both the source of a wave and its speed in a medium from boundary measurements"

Photo-acoustic tomography is an imaging method that combines the high resolution of ultrasound and the high contrast capabilities of electromagnetic waves. In this talk I will first introduce the mathematical model of photo-acoustic tomography. A summary of previous uniqueness results will briefly be discussed when recovering the sound speed and source term from measurements on the boundary. I will next present a new partial uniqueness result for the recovery of the sound speed and a proof of the full recovery of the source term assuming the sound speed is known. This proof relies on the temporal Fourier transform.

9:00 AM - **Tommy Trakoolthai** | Cal Poly Pomona

"A multivariate log-normal moment closure technique for the stochastic predator-prey model"

The deterministic Lotka-Volterra model is a simple predator-prey model that classically portrays the interaction between two species, leading to closed curves in the predator-prey phase plane. Using the probability generating function, we develop a corresponding stochastic version of this model, which has the form of a simple birth-death process. This stochastic model involves the expected values of the populations, which are governed by a system of differential equations almost identical in form to the deterministic system. However, we find that the stochastic model is no longer a closed system. To gain a more intuitive understanding of this model, we turn to a moment closure approximation technique, which captures the main features of the stochastic model. Assuming that the distribution of the two populations is roughly multivariate log-normal, we use a moment closure technique to obtain a closed system of differential equations for the expected values, multiplicative variances, and multiplicative covariance of the populations.

10:30 AM - **Wei-Xiang Feng** | UC Riverside

"On the existence of Buchdahl's stability bound in EiBI gravity"

Recently, Banados and Ferreira have proposed an alternative gravity theory formulated in Palatini formalism, called Eddington inspired Born-Infeld gravity (EiBI). This model is completely equivalent to general relativity (GR) in vacuum, while the highly nonlinear coupling nature between matter and gravity has been shown that the singularity problem in GR is avoided. However, the existence of the corresponding Buchdahl's stability bound (Buchdahl's inequality) is still unclear. In this talk, I attempt to address this issue.

Afternoon Session B | Surge 268

2:00 PM - **Tim McELdowney** | UC Riverside

"Hilbert and Jacobson module equivalence"

In ring theory a Jacobson ring is a ring where every prime ideal is an intersection of maximal ideals. Whereas a Hilbert ring is a commutative ring where every G-ideal is also a maximal ideal. It has been known for over half a century that for commutative rings being Hilbert and Jacobson are equivalent and that they are completely classified in the Noetherian commutative ring setting. Recently Hilbert and Jacobson modules were independently invented on opposite ends of the world. It can be shown that for finitely generated modules being Jacobson and Hilbert are equivalent. Using this new equivalence and the classification result from the Noetherian ring case we would build towards a classification of Hilbert/Jacobson Noetherian f.g. modules.

2:30 PM - **Daniel Cicala** | UC Riveside

"A low key introduction to synthetic differential geometry"

In synthetic mathematics, one focuses on imposing axioms on available types. For synthetic differential geometry (SDG), an axiom scheme is imposed on suitable topoi. One highlight to this approach is that SDG is greater than any single model. In this talk, we introduce the axioms of SDG and the intuitionist logic that makes the axioms work. We then define the basic notions in differential geometry: tangent bundles, lie groups, and vector fields. Finally, we construct a topos in which the SDG axioms hold: the familiar (to algebraic geometers) category of formal duals of C-infinity rings.

3:00 PM - **Jade Master** | UC Riverside

"Chemical reachability in open Petri nets"

This talk will give an informal introduction to Petri nets, open Petri nets, and the category of open Petri nets which will appear in my paper (not on ArXiv yet). I will show how this category can be used in the chemical reachability problem. In particular, there is a functor from the category of open Petri nets to the category of relations which tells you which concentrations are related to each other by chemical reactions.

3:30 PM - **Ryan Ta** | UC Riverside

"Existence of a global solution to the relativistic Vlasov-Poisson equation in dimension three"

We investigate the global existence of solutions to the relativistic Vlasov-Poisson system with non-negative initial data. This is accomplished by converting the question to one of establishing a fixed point for a linear operator acting on a function space. We will address all-time existence of both strong and weak solutions over a three-dimensional Euclidean space.

4:00 PM - **Meijke Balay-Mickelson** | UC Los Angeles

"The Krzyż conjecture and related coefficient problems"

The famous Bieberbach conjecture states that among all analytic and univalent functions on the unit disc $\mathbb{D}$ of the form $f(z) = z + a_2 z^2 + \ldots$, we have $|a_n| \leq n$ with equality for the Koebe function and its rotations. L. de Branges proved the conjecture with the methods of Loewner theory. We consider the related open problem of Krzyż, which states that in the collection $\mathcal{B}$ of analytic functions $f : \mathbb{D} \to \overline{\mathbb{D}} - \{0\}$ we have $\sup\{|f^{(n)}(0)|/n! : f \in \mathcal{B}\} \geq 2/e$ with equality for $f(z) = \exp((z^n - 1)/(z^n + 1))$ and its rotations. We apply the methods of Loewner and Szapiel for the first few coefficients and elucidate the relationship with de Branges' theorem and related coefficient problems.

4:30 PM - **Chon In Luk**, Yoseph Dawit, Tanner Thomas, John Kath, David Perez | Cal Poly Pomona

"Determining the probability of all lattice sample paths of an alternating birth-death chain traveling from state i to state j within a strip"

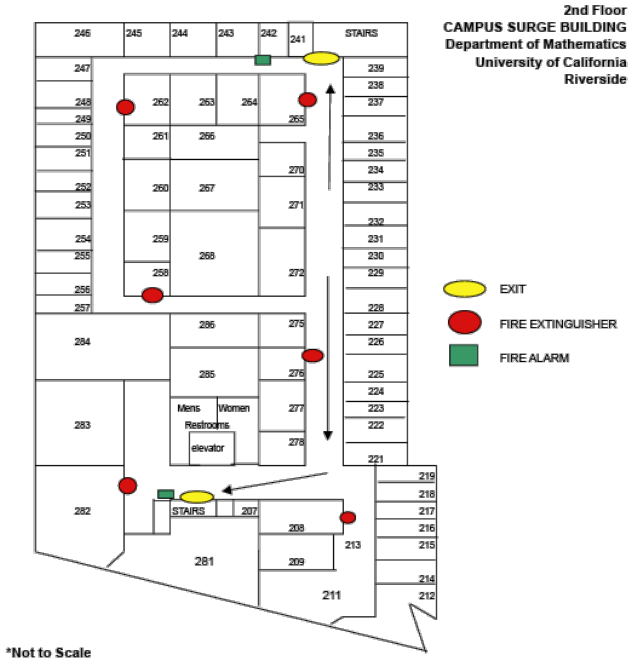
Based upon the 2006 and 2008 articles of Said Kouachi on determining eigenvalues of a certain class of tridiagonal matrices, we are able to find the exact eigenvalues of certain, finite birth-death chains having alternating probabilities for those states restricted to a strip $0 \leq y \leq h$ for any $h \in \mathbb{N}$. As a consequence, we are able to calculate exact expressions for the probability of all lattice paths traveling from state i to state j in n-steps where all states of the lattice path are confined to a strip $0 \leq y \leq h$ for any $h \in N$. Our results depends upon algorithms that use eigenvalues of a matrix M to determine M^n for $n \in \mathbb{N}$. These algorithms have been programmed to make them easier and effective to use. We present and discuss two specific examples that satisfy the different underlying assumptions of Kouachi's 2006 and 2008 articles.

General Information

What is MathConnections?

MathConnections 2018 is a free mathematics conference organized **by graduate students, for graduate students**. Its goal is to bring together graduate students from Southern California and surrounding areas. The conference will consist of 25 minute talks given by graduate students, as well as two plenary talks given by Professor Richard Schoen of UC Irvine, and by Professor Po-Ning Chen of UC Riverside. Each year the conference features a special session on the interplay between two or more areas of math. This year's is on **Geometry and Differential Equations**.

Surge Floorplan



Internet Access:

eduroam

- *Username:* yourID@yourschool.edu
- *Password:* Your normal university password

Organizers:

MathConnections 2018 is being organized by these graduate students: Dane Lawhorne (Lead Organizer), Jolene Britton, Michael McNulty, Joseph Moeller, Dylan Noack, Xavier Ramos Olivé, Mike Pierce, and Alex Pokorny. The organizers are grateful for support from the following sponsors:

AMS American Mathematical Society

GS Graduate Student Association

UCR Department of Mathematics