HW 3
due April 27

1. Prove that if $F \subset K$ is an extension and $[K : F] = 2$, then K is normal over F.
2. Which of the following extensions are normal? Why or why not?
   (a) $\mathbb{Q} \subset \mathbb{Q}(\zeta_n)$, where $\zeta_n = e^{2\pi i/n}$.
   (b) $\mathbb{Q} \subset \mathbb{Q}(\sqrt{2}, \sqrt[3]{2})$.
   (c) $F = \mathbb{F}_3(t) \subset F(\alpha)$, where t is a variable and α is a root of $x^3 - t$ in a splitting field.
3. Let $F \subset K \subset L$ be a tower of field extensions. Prove or disprove:
   (a) If L is normal over F, then K is normal over F.
   (b) If L is normal over F, then L is normal over K.
   (c) If K is normal over F and L is normal over K, then L is normal over F. (If you want a hint, see Exer. 20 of V.3)
4. Let $F \subset K \subset L$ be a tower of field extensions. Show that if L is separable over F, then L is separable over K and K is separable over F. (Remark: it turns out the converse is also true, but we will need a little more machinery before we can prove it.)
5. Suppose that F has characteristic p and $F \subset L$ is a finite extension. Prove that if $p \nmid [L : F]$, then $F \subset L$ is separable.