Math 132 - HW 10
due February 20

February 13, 2015

Unless stated otherwise, we consider $\mathbb{R}^n$ as an inner product space with respect to the dot product.

1. Let w be a non-zero vector in an inner product space V. For an arbitrary vector $v \in V$, express v as a sum of a vector orthogonal of w and a scalar multiple of w. In such a decomposition of v, we call the scalar multiple of w the *orthogonal projection* of v onto w.

2. Apply the Gram-Schmidt process to the given linearly independent subsets of inner product spaces.
   (a) Let $V = \mathbb{R}^3$ and $S = \{(1,0,1),(0,1,1),(1,3,3)\}$.
   (b) Let $V = \mathbb{R}^3$ and $S = \{(1,1,1),(0,1,1),(0,0,1)\}$.
   (c) Let V be the vector space of polynomials over $\mathbb{R}$ of degree at most two, endowed with the inner product $\langle f,g\rangle = \int_0^1 f(t)g(t)\ dt$, and $S = \{1, x, x^2\}$.

3. If $S \subset V$ is a subset of an inner product space, we denote by $S^\perp$ the set
$$\{x \in V | \langle x, s\rangle = 0, \forall s \in S\}.$$
   Prove that:
   (a) $S^\perp$ is a subspace of V.
   (b) If $S_0 \subset S$, then $S^\perp \subset S_0^\perp$.
   (c) $S \subset (S^\perp)^\perp$ and conclude $\mathrm{span}(S) \subset (S^\perp)^\perp$.
   (d) For any finite dimensional subspace $W \subset V$, $W = (W^\perp)^\perp$.