Math 132 - HW 13
due March 4

February 25, 2015

1. True/False - give a counterexample or proof!
   (a) Every self-adjoint operator is normal.
   (b) Operators and their adjoints have the same eigenvectors.
   (c) The identity and zero operators are self-adjoint.
   (d) Every normal operator is diagonalizable.

2. Let T be a linear operator on an inner product space V, and let W be a T-invariant subspace of V. Prove the following:
   (a) If T is self-adjoint, then $T|_W$ is self-adjoint.
   (b) $W^\perp$ is a T^*-invariant.
   (c) If W is both T- and T^*-invariant, then $(T|_W)^* = (T^*)|_W$.
   (d) If W is both T- and T^*-invariant and T is normal, then $T|_W$ is normal.

3. Let T be a normal operator on a finite-dimensional inner product space V. Prove that $\mathrm{Ker}(T) = \mathrm{Ker}(T^*)$ and $\mathrm{Im}(T) = \mathrm{Im}(T^*)$. Hint: for the second part use the equality $\mathrm{Im}(T^*) = \mathrm{Ker}(T)^\perp$ that you were asked to show on the previous homework.