Math 132 - HW 14
due March 6

February 27, 2015

1. Let T be a normal operator on a finite-dimensional complex inner product space V, and let W be a subspace of V. Prove that if W is T-invariant, then W is also T^*-invariant. (Recall you proved on an earlier homework that if $T : V \to V$ is a diagonalizable operator then the restriction of T to any T-invariant subspace is diagonalizable.)

2. A linear operator T on a finite dimensional inner product space V is called *positive definite* if T is self-adjoint and $\langle T(x), x\rangle > 0$ for all $x \neq 0$.
   (a) If T is a self-adjoint operator on a finite dimensional inner product space, prove that T is positive definite if and only all of its eigenvalues are positive.
   (b) Prove that any positive definite operator is invertible and that its inverse is again positive definite.
   (c) Prove that if T is positive definite, then $\langle x, y\rangle_T := \langle T(x), y\rangle$ defines another inner product on V.

3. Let V be a finite-dimensional inner product space, and let T and U be self-adjoint operators on V such that T is positive definite. Prove that both TU and UT are diagonalizable linear operators that have only real eigenvalues. (Hint: to show that UT is diagonalizable and has only real eigenvalues, show that it is self-adjoint with respect to the inner product defined in the previous problem. For TU, repeat the same argument with the inner product $\langle x, y\rangle_{T^{-1}}$.)