Math 132 - HW 15
due March 9

March 2, 2015

1. True/False - give a counterexample or proof!
   (a) Every unitary operator is normal.
   (b) Every orthogonal operator is diagonalizable.
   (c) An operator is unitary if and only if it is invertible.
   (d) The sum of unitary operators is unitary.
   (e) If T is an orthogonal operator on V, then $[T]_\beta$ is an orthogonal matrix for any ordered basis β for V.
   (f) The composition of any two unitary operators is unitary.
   (g) Let U be a linear operator on a finite-dimensional inner product space V and β be an orthonormal basis for V. If $||U(x)|| = ||x||$ for an $x \in \beta$, then U is unitary.
2. For $z \in \mathbb{C}$ define $T_z : \mathbb{C} \to \mathbb{C}$ by $T_z(u) = zu$. For which $z \in \mathbb{C}$ is T_z normal? self-adjoint? unitary?
3. Let U be a unitary operator on an inner product space V, and let W be a finite-dimensional U-invariant subspace of V. Prove that
   (a) $U(W) = W$,
   (b) $W^\perp$ is U-invariant.