Math 132 - HW 3
due January 16

January 9, 2015

1. For each matrix A and ordered basis β, find $[L_A]_\beta$. Here L_A denotes the linear operator on $\mathbb{R}^n$ defined by A and $[L_A]_\beta$ denotes the matrix representation of the linear operator L_A in the ordered basis β.

   (a)
$$A = \begin{pmatrix} 1 & 3 \\ 1 & 1 \end{pmatrix}, \beta = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$$

   (b)
$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \beta = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}.$$

2. For each matrix $A \in \mathrm{Mat}_{n\times n}(\mathbb{C})$ given below, determine the eigenvalues and the set of eigenvectors corresponding to each eigenvalue. Then in each case, find a basis for $\mathbb{C}^2$ consisting of eigenvectors. Finally, determine an invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ = D$.

   (a)
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$$

   (b)
$$B = \begin{pmatrix} i & 1 \\ 2 & -i \end{pmatrix}.$$

3. Let T be a linear operator on a vector space V.

   (a) Suppose T is invertible and prove that a scalar λ is an eigenvalue of T if and only if λ^{-1} is an eigenvalue of T^{-1}.

   (b) Suppose that x is an eigenvector of T corresponding to the eigenvalue λ. For any positive integer m, prove that x is an eigenvector of T^m corresponding to the eigenvalue λ^m.