Math 132 - HW 4
due January 21

January 14, 2015

1. (a) What is the definition of a diagonalizable linear operator T on a finite dimensional vector space V?

   (b) What is the definition of a diagonalizable $n \times n$-matrix?

   (c) For each of the following matrices $A \in \mathrm{Mat}_{n\times n}(\mathbb{R})$, determine if A is diagonalizable (over $\mathbb{R}$), and if it is diagonalizable, find a matrix Q such that $Q^{-1}AQ$ is a diagonal matrix.

$$(a)\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, \qquad (b)\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}, \qquad (c)\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$$

$$(d)\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}, \qquad (e)\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

2. True/False: Determine which of the following statements are true. For each true statement, give a proof and for each false statement produce a counterexample.

   (a) Any linear operator on an n-dimensional vector space that has fewer than n distinct eigenvalues is not diagonalizable.

   (b) Eigenvectors corresponding to the same eigenvalue are always linearly dependent.

   (c) If λ is an eigenvalue of a linear operator T, then each element of the eigenspace E_λ is an eigenvector of T.

   (d) If λ_1 and λ_2 are distinct eigenvalues of a linear operator T, then $E_{\lambda_1} \cap E_{\lambda_2} = \{0\}$.

   (e) Let $A \in M_{n\times n}(\mathbb{F})$ and $\beta = \{v_1, \ldots, v_n\}$ be an ordered basis for $\mathbb{F}^n$ consisting of eigenvectors of A. If Q is the $n \times n$ matrix whose i-th column is v_i $(1 \leq i \leq n)$, then $Q^{-1}AQ$ is a diagonal matrix.