Math 132 - HW 5
due January 23

January 16, 2015

1. True/False: Determine which of the following statements are true. For each true statement, give a proof and for each false statement produce a counterexample.
   (a) A linear operator T on a finite-dimensional vector space is diagonalizable if and only if the multiplicity of each eigenvalue λ equals the dimension of E_λ.
   (b) Every diagonalizable linear operator on a nonzero vector space has at least one eigenvalue.

2. (a) Let D be a diagonal matrix:
   $$D = \begin{pmatrix} d_1 & 0 & \dots & 0 \\ 0 & d_2 & & \\ \vdots & & \ddots & \vdots \\ 0 & & \dots & d_n \end{pmatrix}$$
   Compute D^n.
   (b) For
   $$A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix},$$
   find an expression for A^n, where n is an arbitrary positive integer. (Hint: If a matrix A is diagonalizable, then there is an invertible matrix Q such that $Q^{-1}AQ$ is a diagonal matrix D, or in other words $A = QDQ^{-1}$...)

3. Two linear operators T and U on a finite-dimensional vector space V are called **simultaneously diagonalizable** if there exits an ordered basis β for V such that both $[T]_\beta$ and $[U]_\beta$ are diagonal matrices. Prove that if T and U are simultaneously diagonalizable operators, then T and U commute (i.e. $TU = UT$).