Math 132 - HW 7
due January 30

January 23, 2015

1. Let T be a diagonalizable linear operator on a finite-dimensional vector space V, and let W be a T-invariant subspace of V. Suppose that $v_1, v_2, \ldots, v_k$ are eigenvectors of T corresponding to distinct eigenvalues. Prove that if $v_1 + v_2 + \ldots v_k$ is in W, then $v_i \in W$ for all i. (Hint: Use induction on k and the trick we used to show that eigenvectors with distinct eigenvalues are linearly independent.)

2. Prove that the restriction of a diagonalizable linear operator T to any nontrivial T-invariant subspace is also diagonalizable. (Hint: Use the result from the previous problem.)

3. Prove that if T and U are diagonalizable linear operators on a finite-dimensional vector space V such that $UT = TU$, then T and U are simultaneously diagonalizable. (See problem 3 of homework 5 for the definition of simultaneously diagonalizable.) Hint: For any eigenvalue λ of T show that the eigenspace E_λ is U-invariant, and then apply the previous problem to obtain a basis for E_λ made up of eigenvectors for U.