Math 132 - HW 9
due February 18

February 11, 2015

Midterm corrections: If you did worse than you would have liked to do on the midterm, I invite you to write solutions to the problems you missed (or received partial credit for) and turn them in at the beginning of class on Wednesday Feb. 18th, attached by staple to your original test. I will grade them and give you up to 50% credit for your corrections. For example, if you missed 16 points on the exam, and you write up (your own!) correct and well-written solutions to all of the problems you missed or got partial credit on, I will add 8 points to your original score.

1. Consider the vector space $M = \mathrm{Mat}_{n\times n}(\mathbb{R})$ of real $n \times n$-matrices. For any two matrices $A, B \in M$, let
$$\langle A, B\rangle = \mathrm{tr}(B^t A).$$
   (a) Prove that this defines an inner product structure on the vector space M.
   (b) Compute the norm of the identity matrix $I \in \mathrm{Mat}_{n\times n}(\mathbb{R})$ with respect to the inner product structure defined above.
   (c) For any integers i and j such that $1 \leq i, j \leq n$, let $E_{i,j}$ denote the matrix with all zero entries except for a one in the (i, j)-th entry. Show that the matrices $E_{i,j}$ form an orthonormal basis with respect to the above inner product structure.

2. For each of the following pairings determine whether or not it defines an inner product structure and explain why or why not.
   (a) $\langle (a, b, c), (e, f, g)\rangle = ae + 3bf - cg$ for the real vector space $\mathbb{R}^3$,
   (b) $\langle (a, b), (c, d)\rangle = ac + ad + bc + 3bd$ for the real vector space $\mathbb{R}^2$,

3. Let V be an inner product space. For any $x, y \in V$, show that
$$||x + y||^2 + ||x - y||^2 = 2||x||^2 + 2||y||^2.$$