Math 132 - HW 10

1. For each of the following, either prove that it defines an inner product on the given vector space or provide a reason why it does not.

   (a) $\langle (a, b), (c, d) \rangle = ac - bd$ on $\mathbb{R}^2$.

   (b) $\langle A, B \rangle = \operatorname{tr}(A + B)$ on $\mathrm{Mat}_{2\times 2}(\mathbb{R})$. (Recall that the trace $\operatorname{tr}(A)$ of an $n \times n$-matrix is the sum of its diagonal entries.)

   (c) $\langle p, q \rangle = \int_0^1 p'(t)q(t)dt$ on $\mathcal{P}(\mathbb{R})$.

2. Suppose V is an inner product space, $T \in \mathcal{L}(V)$ is a linear operator and that for any $v \in V$,
$$||T(v)|| \leq 2||v||.$$
   Prove that $T - 3I$ is invertible.