Math 132 - HW 11

1. Let $V = C([-1,1])$ be the real vector space of all continuous functions $f : [-1,1] \to \mathbb{R}$. We think of V as an inner product space by endowing it with the inner product

$$\langle f, g \rangle = \int_{-1}^{1} f(t)g(t)\ dt.$$

   Note that we can consider $1, 1+t, t^2$ as vectors in V.

   (a) Prove that the list $1, 1+t, t^2$ is linearly independent.

   (b) Apply the Gram-Schmidt process to the list $1, 1+t, t^2$ to obtain an orthonormal list.

2. Let β be the basis $(3,0,0), (2,2,2), (1,1,2)$ of $\mathbb{R}^3$ (you might want to check for yourself that this is indeed a basis). Suppose that $T \in \mathcal{L}(\mathbb{R}^3)$ and that $\mathcal{M}(T, \beta)$ is upper-triangular. Find an orthonormal basis γ of $\mathbb{R}^3$ such that $\mathcal{M}(T, \gamma)$ is still upper triangular.