Math 132 - HW 12

As we didn't finish talking about the Riesz representation theorem today, the following problems are instead on some background material to help you brush up.

1. Let V be a vector space of dimension n and fix a basis $\beta = (v_1, \ldots, v_n)$. Recall that the set of all linear operators $T \in \mathcal{L}(V)$ is a vector space. Today we recalled how to associate a matrix $\mathcal{M}(T, \beta)$ to T with respect to the basis β. Consider the map $\mathcal{M} : \mathcal{L}(V) \to \mathrm{Mat}_{n\times n}(\mathbb{F})$ that takes a linear operator T to the matrix $\mathcal{M}(T, \beta)$.
   (a) Prove that $\mathcal{M}$ is linear.
   (b) Prove that $\mathcal{M}$ is an isomorphism of vector spaces and deduce that $\dim \mathcal{L}(V) = n^2$.

2. Let V be a vector space of dimension n and $U \subset V$ a subspace of dimension k. Consider the subset $L \subset \mathcal{L}(V)$ consisting of all linear operators $T : V \to V$ such that U is a T-invariant subspace.
   (a) Prove that L is in fact a subspace of $\mathcal{L}(V)$.
   (b) Compute (with proof) the dimension of L. (Hint: compare with the previous problem)