Math 132 - HW 13

1. Recall from Homework 11 the following set-up: Consider $V = C([-1,1])$ as an inner product space with the inner product

$$\langle f, g\rangle = \int_{-1}^{1} f(t)g(t)\ dt.$$

In that homework problem, you were asked to apply the Gram-Schmidt procedure to the list $1, x+1, x^2$. Hopefully you obtained as a result the orthonormal list:

$$\sqrt{\frac{1}{2}}, \sqrt{\frac{3}{2}}x, \sqrt{\frac{45}{8}}(x^2 - 1/3).$$

Note that we can consider $\mathcal{P}_2(\mathbb{R})$ as a subspace of V, thus as an inner product space with respect to the same inner product. Moreover, the orthonormal list above is in fact an orthonormal basis of $\mathcal{P}_2(\mathbb{R})$.

   (a) Let $\phi : \mathcal{P}_2(\mathbb{R}) \to \mathbb{R}$ be the map that takes a polynomial p to its value at $1/3$ (i.e., $p \mapsto p(1/3)$). Show that ϕ is a linear functional on $\mathcal{P}_2(\mathbb{R})$.

   (b) Find a polynomial $q \in \mathcal{P}_2(\mathbb{R})$ such that $p(1/3) = \langle p, q\rangle$ for any $p \in \mathcal{P}_2(\mathbb{R})$.

2. Let U and W be subspaces of an inner product space V. Prove that $(U+W)^\perp = U^\perp \cap W^\perp$.