

## Math 132 - HW 14

1. Suppose that  $U$  and  $W$  are finite-dimensional subspaces of an inner product space  $V$ . Prove that  $P_{W^\perp}P_U = 0$  if and only if  $U \subset W$ .
2. Let  $V$  be a finite-dimensional inner product space and let  $P \in \mathcal{L}(V)$  be a linear operator such that  $P^2 = P$ . Prove that there exists a subspace  $U$  of  $V$  such that  $P = P_U$  if and only if  $\text{null}(P) = (\text{range}(P))^\perp$ .