Math 132 - HW 17

1. True/False - give a counterexample or proof!
   (a) Every normal operator is self-adjoint.
   (b) Operators and their adjoints have the same eigenvectors.
   (c) The identity and zero operators are self-adjoint.
   (d) Every normal operator is diagonalizable.
2. Let T be a linear operator on an inner product space V, and let W be a T-invariant subspace of V. Prove the following:
   (a) If T is self-adjoint, then $T|_W$ is self-adjoint. [1]
   (b) If W is both T- and T^*-invariant, then $(T|_W)^* = (T^*)|_W$.
   (c) If W is both T- and T^*-invariant and T is normal, then $T|_W$ is normal.
3. Let T be a normal operator on a finite-dimensional inner product space V. Prove that $\text{null}(T) = \text{null}(T^*)$ and $\text{range}(T) = \text{range}(T^*)$. Hint: for the second part recall that $\text{range}(T^*) = \text{null}(T)^\perp$ and similarly with T replaced by T^*.

[1] Here$T|_W : W \to W$ denotes the linear operator defined by $T|_W(w) = T(w)$ for any $w \in W$.