Math 132 - HW 18

1. Let T be a diagonalizable linear operator on a finite-dimensional vector space V, and let W be a T-invariant subspace of V. Suppose that $v_1, v_2, \ldots, v_k$ are eigenvectors of T corresponding to distinct eigenvalues. Prove that if $v_1 + v_2 + \cdots + v_k$ is in W, then $v_i \in W$ for all i. (Hint: Use induction on k and the trick we used to show that eigenvectors with distinct eigenvalues are linearly independent.)

2. Prove that the restriction of a diagonalizable linear operator T to any nontrivial T-invariant subspace is also diagonalizable. (Hint: Use the result from the previous problem.)

3. Let T be a normal operator on a finite-dimensional complex inner product space V, and let W be a subspace of V. Prove that if W is T-invariant, then W is also T^*-invariant. (Hint: Use the result from the previous problem.)