1. Prove that a group is isomorphic to itself.

2. Prove that $\mathbb{Z}_n \cong \mathbb{Z}_m$ if and only if $m = n$.

3. Recall the Klein 4-group $K$ given by the table

\[
\begin{array}{c|cccc}
* & e & a & b & c \\
e & e & a & b & c \\
a & a & e & c & b \\
b & b & c & e & a \\
c & c & b & a & e \\
\end{array}
\]

Prove that $K$ is not isomorphic to $\mathbb{Z}_4$.

4. Prove that $(\mathbb{R}, +) \cong (\mathbb{R}_{>0}, \cdot)$.

5. Recall that the kernel of a homomorphism $f : G \to H$ is defined as

$$\ker(f) = \{ g \in G : f(g) = e_H \}$$

Prove that $\ker(f)$ is a subgroup of $G$.

6. Prove that a homomorphism is injective if and only if its kernel is trivial.

7. Consider the map $f : \mathbb{Z} \to \mathbb{Z}_5$ where $f(m)$ is defined to be the remainder when $m$ is divided by 5. More precisely, we can write any integer as $m = 5q + r$ where $q \in \mathbb{Z}$ and $r \in \mathbb{Z}_5$; we define $f(m) = r$.

   (a) Prove that this map is a surjective homomorphism.

   (b) Prove that $\ker(f)$ is the set of integers divisible by 5.

8. Let $f : G \to H$ be a homomorphism and $h \in \ker(f)$. Prove that $ghg^{-1} \in \ker(f)$ for any $g \in G$. 