Homework Zero

Integral Calculus for Life Sciences

UCR Math-007B-B01, Summer 2019

The purpose of this homework is to give you a review of some mathematics that I think you've seen before that will come up in this course. So if there is anything in this homework that doesn't look familiar, or that *does* look familiar but you find yourself struggling with, you should let me know so that I can teach it to you, and maybe to the rest of the class too. Remember that I have office hours on Mondays after lecture, and that my email address is mpierce@math.ucr.edu.

1. Read over the syllabus. that I posted to the course webpage. If you have any questions about the class, please ask me via email or in the first day of lecture.

2. For each of the following equations, describe the set of points in the (x, y)-plane that satisfy that equation. (Note that a picture might serve very well as a description)

$y - \ln(x) = 0$	$x^2 + y^2 = 5$	$x^2 - y^2 = 2$
$x = y^2 - 5$	$9x = 0$	$x^2 + 3y^2 = 1$
$(x-2)^2 + (y+6)^2 = 1$	$xy - 1 = 0$	$y = \dfrac{x-3}{(x+1)(x-4)^2}$

3. Recall that the definition of the absolute value function is given by

$$|x| := \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}.$$

Now, either algebraically or geometrically, can you find all values of x that satisfy each of these inequalities?

$\lvert 3x+4\rvert > 5$	$\lvert x^2\rvert \leq 3$	$\lvert\lvert 3x+7\rvert - 5\rvert < 2$

4. (Even and Odd Functions) Recall that a function f is even if $f(-x) = f(x)$, and that a function f is odd if $f(-x) = -f(x)$.

   (a) How can you tell whether a function is even or odd by looking at its graph?

   (b) Write down many examples of even functions until you are satisfied.

   (c) Write down many examples of odd functions until you are satisfied.

(d) Suppose that f and g are both even functions. Can you say if fg must be even or odd? What can you say about fg if f and g are both odd? What can you say about fg if f is even and g is odd?

(e) Like in the last part, investigate the even-ness or odd-ness of the function $f+g$ in the three cases: both f and g are even, both f and g are odd, and one of f and g is even and the other is odd.

(f) Can you *prove* that the only function that is both even and odd is $f(x) = 0$?

5. What is the definition of the derivative of a function f? (HINT: remember this from your last calculus class; it involves a limit)

6. What is the derivative of each of the following functions? I'll post what I got for their derivatives at the end of this document.

$a(x) = \frac{3x^3+1}{2x+3}$ $\qquad$ $b(x) = e^x + e^\pi$

$f(x) = 4x^3 + \pi x^2 - 1$ $\qquad$ $g(x) = \sec(x)\tan(x)$

$h(x) = \sin(x)\cos(x)$ $\qquad$ $p(x) = \frac{\csc(3x)+\cos(x)}{x^3}$

$q(x) = 2\ln\left(\sin(x^2)\right)$ $\qquad$ $r(x) = x^\pi e^{-9x^2}$

7. (PARTIAL FRACTION DECOMPOSITION) I suspect that some of you might *not* have seen partial fraction decomposition before. If this is the case, let me know in the first lecture so I know to spend more time on it when it comes up in the course. If you have learned this before, to test your skills, give me the partial fraction decompositions of the following rational expressions.

$$\frac{2x-3}{(x-1)(x-2)} \qquad \frac{x-12}{3x^2+3x-18} \qquad \frac{3x^3+7x^2+5x+3}{(x+1)^2(x^2+1)}$$

8. (ARITHMETIC) I don't know how to make this an exercise, so let me just show you this trick. Notice that

$$\frac{x}{x+1} = \frac{x+(1-1)}{x+1} = \frac{(x+1)-1}{x+1} = \frac{x+1}{x+1} + \frac{-1}{x+1} = 1 - \frac{1}{x+1}.$$

This can be useful because it takes an expression with *two* xs and writes it as an expression with only one x. You can intuitively see that these are equal since each represents "one part less than a whole." For example, letting x be specific integers, this equivalence is just

$$\frac{7}{8} = 1 - \frac{1}{8} \qquad \text{or} \qquad \frac{100}{101} = 1 - \frac{1}{101} \qquad \text{or...}$$

9. (TRIGONOMETRY) You're going to have to remember some trigonometry for this course. In particular you'll need to know the "unit circle" (like, can you tell me what $\sin(5\pi/6)$ is? Or $\cot(11\pi/3)$? Or what about $\text{arcsec}(2)$?), and you'll have to know the Pythagorean identities and the double-angle and half-angle formulas (I don't think the sum- or difference-of-angles formulas will come up though). Below, I'm going to work you through how I remember the Pythagorean identities and the double-angle and half-angle formulas. Or you could just ignore this exercise, look them up and memorize them.

   Recall our favorite trigonometric identity, the main Pythagorean identity

$$\cos^2(\theta) + \sin^2(\theta) = 1^2 .$$

   This formula is true because it's just the Pythagorean theorem applied to a triangle in the unit circle with hypotenuse 1, base $\cos(\theta)$, and height $\sin(\theta)$.

   (a) Write down the two other Pythagorean identities that involve $\csc^2$, $\sec^2$, $\tan^2$, and $\cot^2$. You can do this by taking the main Pythagorean identity and dividing everything by $\sin^2(\theta)$ to get one new identity, and by dividing everything by $\cos^2(\theta)$ to get the other identity.

   (b) Let's figure out the double-angle formulas too. The way I like to do this is by having one very nice formula committed to memory called De Moivre's formula:

$$\big(\cos(\theta) + i\sin(\theta)\big)^n = \cos(n\theta) + i\sin(n\theta)$$

   where $i = \sqrt{-1}$. I find this one formula easier to remember than the individual double-angle formulas. Now for $n = 2$ we get a less general formula $\big(\cos(\theta) + i\sin(\theta)\big)^2 = \cos(2\theta) + i\sin(2\theta)$. Multiply out the left-hand side of this equation, and compare the real/imaginary parts on the left with the real/imaginary parts on the right to get the double-angle formulas

$$\cos^2(\theta) - \sin^2(\theta) = \cos(2\theta)$$
$$2\sin(\theta)\cos(\theta) = \sin(2\theta) .$$

   (c) Then since we have the double-angle formulas we can get the half-angle formulas now. Each of these formulas above are true *for all* θ that you could plug into them. This means that they are true for $\frac{1}{2}\theta$ too. Let's write the main Pythagorean identity and the double-angle formula for cosine in terms of $\frac{1}{2}\theta$:

$$\cos^2\left(\frac{1}{2}\theta\right)+\sin^2\left(\frac{1}{2}\theta\right)=1 \qquad \text{and} \qquad \cos^2\left(\frac{1}{2}\theta\right)-\sin^2\left(\frac{1}{2}\theta\right)=\cos(\theta)$$

Add these two equations together, and solve for $\cos\left(\frac{1}{2}\theta\right)$ to get the half-angle formula for cosine. Then subtract these two equations, and solve for $\sin\left(\frac{1}{2}\theta\right)$ to get the half-angle formula for sine.

10. Evaluate these composite functions on the specified input. Give the output exactly please: you don't need to bother with decimal approximations. I'll put my answers at the end of this document.

    $\sin\left(\arcsin\left(\frac{7}{9}\right)\right)$ $\qquad$ $\arccos\left(\cos\left(\frac{5\pi}{3}\right)\right)$

    $\sin\left(\arctan(42)\right)$ $\qquad$ $\csc\left(\arccos\left(\frac{1}{2}\right)\right)$

11. (SIGMA SUMMATION NOTATION) You might not have seen this before, so let me explain it at least a little bit now, and maybe more in lecture. Suppose you're looking at the sum of the first seventeen squares:

    $$1+4+9+16+25+36+49+64+81+100+121+144+169+196+225+256+289$$

    That's long, and sucks to write out. Instead of writing out long sums, mathematicians commonly use a more terse notation, sigma notation:

    $$\sum_{i=\text{start}}^{\text{end}} \text{summand formula}.$$

    So that sum of the first seventeen squares can be written as $\sum_{n=1}^{17} n^2$. As an exercise in understanding this notation, figure out the values of the following sums. I've put my answers at the end of this document.

    $\sum_{n=0}^{13} n$ $\qquad$ $\sum_{i=2}^{5} i^3$ $\qquad$ $\sum_{k=-3}^{4} 4$

    $\sum_{n=0}^{1001} (-1)^n$ $\qquad$ $\sum_{k=4}^{7} \frac{1}{k-2}$ $\qquad$ (TOUGHER) $\sum_{i=1}^{\infty} 2^{-i}$

My solutions to Exercise 6

$$\frac{12x^3+27x^2-2}{(2x+3)^2}$$

$$e^x$$

$$12x^2+2\pi x$$

$$2\sec^3(x)-\sec(x)$$

$$2\cos^2(x)-1$$

$$-3x^{-3}\big(\csc(3x)\cot(3x)+\sin(x)\big)-3x^{-4}\big(\csc(3x)+\cos(x)\big)$$

$$4x\cot\left(x^2\right)$$

$$\left(\pi x^{\pi-1}-18x^{\pi+1}\right)e^{-9x^2}$$

My solutions to Exercise 10

$\frac{7}{9}$	$\frac{\pi}{3}$
$\frac{42}{\sqrt{1765}}$	$\frac{2}{\sqrt{3}}$

My solutions to Exercise 11

91	224	32
0	$\frac{77}{60}$	1