Assessment One

Integral Calculus for Life Sciences
UCR Math-007B-B01, Summer 2019

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This assessment is intended not only as a way to determine if you've understood what you were asked to think about for the homework, but also as a means for you to assess your own understanding of that material, and assess if you're meeting your own expectations for yourself. I expect that it'll be challenging for anyone to respond to all these prompts in the allotted time, but that's okay; it wouldn't be a very useful assessment if it wasn't adequately challenging.

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1. What is the definition of the derivative of a function f? (HINT: it involves a limit)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{OR} \quad \lim_{x_0 \to x} \frac{f(x_0) - f(x)}{x_0 - x}$$

2. What is the double-angle formula for sine?

$$\cos(2\theta) + i\sin(2\theta) = (\cos\theta + i\sin\theta)^2$$

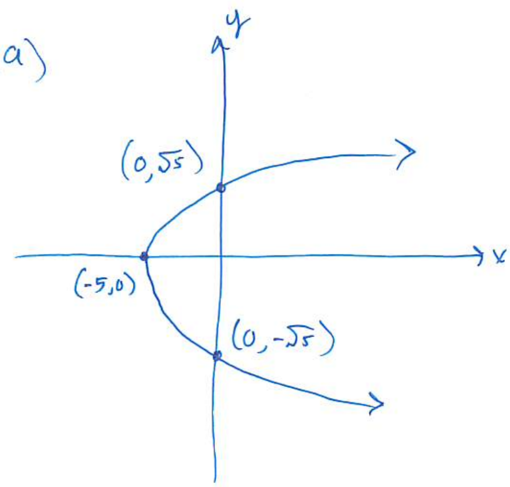
$$= \cos^2\theta - \sin^2\theta + i\,2\cos\theta\sin\theta$$

$$\Longrightarrow \sin(2\theta) = 2\cos(\theta)\sin(\theta) \;//$$

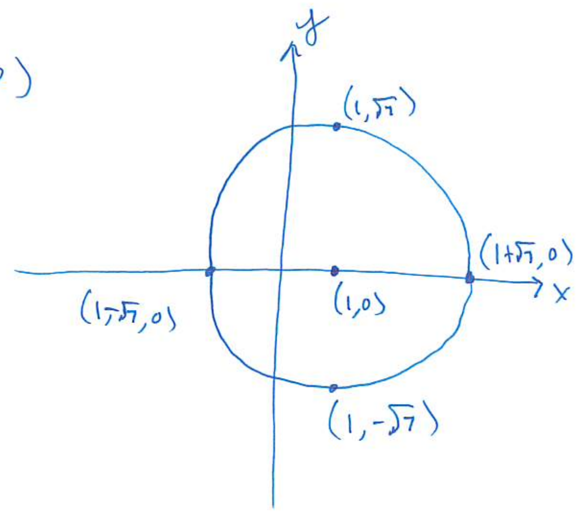
3. For each of the following equations, draw the set of points in the (x,y)-plane that satisfy that equation. Be sure to indicate the position of any interesting features of the set of points, like asymptotes and axis-intercepts and stuff. You should do each on a separate set of axis.

(a) $x = y^2 - 5$ (b) $(x-1)^2 + y^2 = 7$ (c) $x^2 y = 1$ (d) $|x^3 + 3| - y = 0$

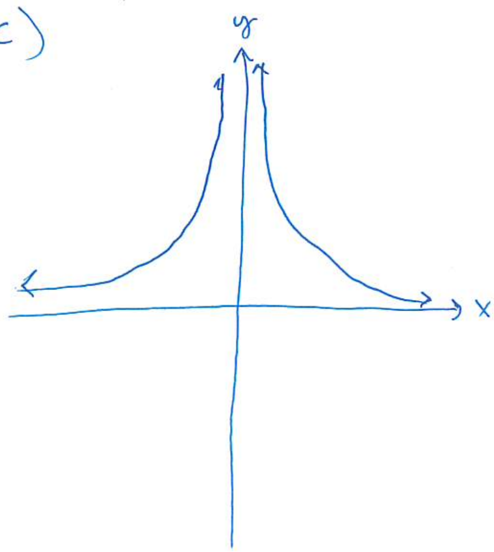
(a)



(b)

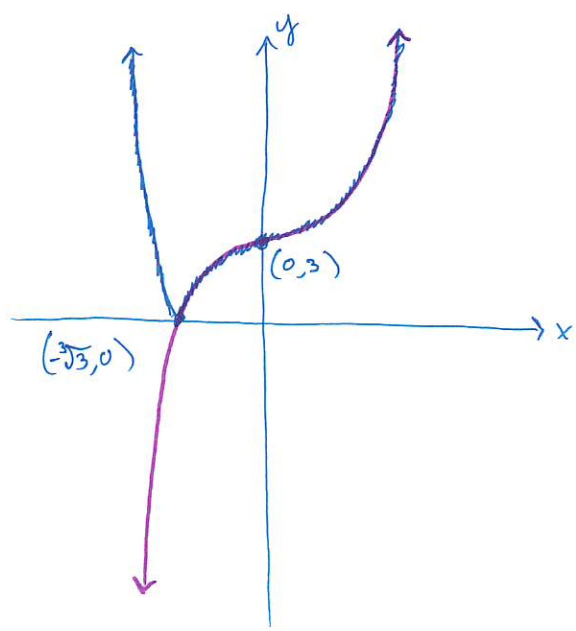


(c)



$x^2 y = 1 \Rightarrow y = \frac{1}{x^2}$

(d) $y = |x^3 + 3|$ $y = x^3 + 3$



4. Below are a bunch of functions of a variable x. Some are even functions, some are odd functions, and some are neither even nor odd. Put a box around the even ones, put a circle around the odd ones, and cross out ones that are neither even nor odd.

$\boxed{f(x) = x^2}$	$f(x) = \sin(x)$	$\boxed{f(x) = \sin^2(x)}$	$\boxed{f(x) = \sin(x^2)}$
$f(x) = x^2 \sin(x)$	$\cancel{f(x) = x^2 + \sin(x)}$	$f(x) = x^3$	$\boxed{f(x) = \cos(x)}$
$f(x) = \cos^3(x)$	$\boxed{f(x) = \cos(x^3)}$	$f(x) = x^3 \cos(x)$	$\cancel{f(x) = x^3 + \cos(x)}$
$\boxed{f(x) = 1}$	$f(x) = x$	$\cancel{f(x) = \sqrt{x}}$	$\boxed{f(x) = \sqrt{1 - x^2}}$
$\cancel{f(x) = e^x}$	$\cancel{f(x) = \ln(x)}$	$\boxed{f(x) = \lvert x \rvert}$	$f(x) = \frac{1}{x}$

f is even if $f(x) = f(-x)$ (symmetric about the y-axis)

f is odd if $f(-x) = -f(x)$ (symmetric about a 180° rotation around the origin)

(even) + (even) = even

(odd) + (odd) = odd

(even) + (odd) = ?

(even)(even) = even

(odd)(odd) = even

(even)(odd) = odd

$\sin((-x)^2) = \sin(x^2)$

$\cos((-x)^3) = \cos(-x^3) = \cos(x^3)$

5. What is the derivative of each of the following functions?

(a) $f(x) = 3\cos\left(x^3\right)\ln\left(\sec(x)\right)$

$$3\left(\cos(x^3)\frac{1}{\sec(x)}\sec(x)\tan(x) + \left(-\sin(x^3)\right)3x^2\ln(\sec(x))\right)$$

$$= 3\left(\cos(x^3)\tan(x) - 3x^2\sin(x^3)\ln(\sec(x))\right) \mathbin{/\!/}$$

(b) $g(x) = \dfrac{2x-11}{3x^2+3x-18}$

$$\frac{2(3x^2+3x-18) - (2x-11)(9x+3)}{(3x^2+3x-18)^2}$$

$$= -12x^2+99x-3 \big/ (3x^2+3x-18)^2 \mathbin{/\!/}$$

(c) $h(x) = \dfrac{1}{2}\tan\left(\ln\left(x^2\right)\right)$

$$\frac{1}{2}\sec^2(\ln(x^2))\left(\frac{1}{x^2}\right)(2x)$$

$$= \frac{\sec^2(\ln(x^2))}{x} \mathbin{/\!/}$$

(d) $q(x) = x^{e}e^{x} + e^{e}$

$$ex^{e-1}e^x + x^e e^x \mathbin{/\!/}$$

6. Tell me the real number that each of the following is equivalent to. Write that real number as nicely as you can, and give the exact value please: you don't need to bother with decimal approximations.

(a) $\ln(1)$

0

(b) $\ln(e)$

1

(c) $\ln\left(e^2\right)$

$2\ln(e) = 2$

(d) $\cos(0)$

1

(e) $\cos\left(-\frac{\pi}{2}\right)$

0

(f) $\tan\left(\frac{\pi}{4}\right)$

1

(g) $\sin\left(\frac{5\pi}{6}\right)$

$1/2$

(h) $\cot\left(\frac{11\pi}{3}\right)$

$= \dfrac{\cos(11\pi/3)}{\sin(11\pi/3)} = \dfrac{\cos 5\pi/3}{\sin 5\pi/3} = \cdots$

$\cdots = \dfrac{1/2}{\sqrt{3}/2} = 1/\sqrt{3}$ //

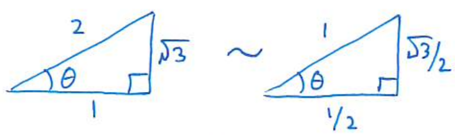
(i) $\sec\left(\frac{-3\pi}{4}\right) = \dfrac{1}{\cos(-3\pi/4)}$

$= \dfrac{1}{\cos(3\pi/4)} = -\sqrt{2}$ //

(j) $\arctan(1)$

$\pi/4$ //

(k) $\operatorname{arcsec}(2) = \theta$

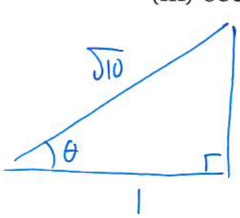


$\theta = \pi/3$ //

(l) $\arctan\left(\tan\left(\frac{7\pi}{22}\right)\right)$

$7\pi/22$

(m) $\sec(\arctan(3))$



$\sqrt{10}$ //

(n) $\sum_{n=1}^{6} 2n$

$2(1)+2(2)+2(3)+\cdots$

$\cdots + 2(4)+2(5)+2(6)$

$= 2\cdot 7\cdot 3 = 42$ //

(o) $\sum_{n=0}^{1002} (-1)^n = (-1)^0 + \cdots + (-1)^{1002}$

$= \underbrace{1 + -1}_{0} + \cdots 1$

$= 1$ //

7. (RECREATIONAL) Suppose that a, b, $a+b$, and $a-b$ are all positive prime numbers. What must the sum of all four of these numbers be?

8. (RECREATIONAL) Suppose you have an 8×8 grid of squares. Remove two of the corner squares that are in diagonally opposite corners. Is it possible to cover the 62 remaining squares in the grid by dominoes (rectangles of size 2×1)?

You should discuss these ones with me. -Mike