Homework One

Integral Calculus for Life Sciences

UCR Math-007B-B01, Summer 2019

1. By appealing to the geometry of the graph of a function f, be able to explain why each of the following equations hold for any real numbers a,b, and c.

$$\int_a^b cf(x)\,\mathrm{d}x = c\int_a^b f(x)\,\mathrm{d}x$$

$$\int_a^b f(x)\,\mathrm{d}x = -\int_b^a f(x)\,\mathrm{d}x$$

$$\int_a^b f(x)\,\mathrm{d}x = \int_a^0 f(x)\,\mathrm{d}x + \int_0^b f(x)\,\mathrm{d}x$$

$$\int_a^b \big(f(x)+g(x)\big)\,\mathrm{d}x = \int_a^b f(x)\,\mathrm{d}x + \int_a^b g(x)\,\mathrm{d}x$$

2. (GEOMETRY) Evaluate the following integrals by looking at the graphs of the integrands and using your knowledge of geometry. My own answers can be found at the end of this document. Note that $\lfloor y \rfloor$ denotes the floor function, and returns the greatest integer less than or equal to y.

$$\int_0^4 (x+1)\,\mathrm{d}x \qquad \int_{-2}^3 (2x-3)\,\mathrm{d}x \qquad \int_{-2}^2 \sqrt{4-x^2}\,\mathrm{d}x$$

$$\int_0^{4\pi} \sin(t)\,\mathrm{d}t \qquad \int_{-4}^{1/3} |3x+4|-5\,\mathrm{d}x \qquad \int_1^5 3\lfloor y \rfloor\,\mathrm{d}y$$

3. (GEOMETRY) Suppose that I tell you that $\int_0^3 x^2\,\mathrm{d}x = 9$. Using geometric methods, what is the value of $\int_0^{\mathbf{9}} \sqrt{x}\,\mathrm{d}x$?

4. (GEOMETRY) Remember that a function f is odd if $f(-x) = -f(x)$. Taking an integral of an odd function f can be very nice because $\int_{-a}^a f(x)\,\mathrm{d}x = 0$. First convince yourself that this is true, then evaluate the following integrals.

$$\int_{-2}^2 t^3\,\mathrm{d}t \qquad \int_{-7}^7 \sin(\theta)\cos(\theta)\,\mathrm{d}\theta \qquad \int_2^4 \tan(\mu-3)\,\mathrm{d}\mu$$

5. (CHALLENGE) Evaluate these integrals.

$$\int_0^{2\pi} \sin(3z)\sin(5z)\sin(7z)\,\mathrm{d}z \qquad \int_{-1}^1 \left(y^5+3\right)\sqrt{1-y^2}\,\mathrm{d}y$$

6. (Riemann Sums) These questions ask you to approximate the value of some integrals using various types of Riemann sums.

   (a) Approximate the value of $\int_0^4 \frac{1}{2}x^3 \, \mathrm{d}x$ by subdividing the domain of integration into four subintervals and using a left-endpoint Riemann sum.

   (b) Approximate the value of $\int_{-3}^{4} -t^2 + 2 \, \mathrm{d}t$ by subdividing the domain of integration into seven subintervals and using a right-endpoint Riemann sum.

   (c) Approximate the value of $\int_{-1}^{5} -t^2 + 2 \, \mathrm{d}t$ by subdividing the domain of integration into *three* subintervals and using a midpoint Riemann sum.

   (d) Write down a right-endpoint Riemann sum that approximates the value of $\int_0^4 \frac{1}{2}x^3 \, \mathrm{d}x$ by subdividing the domain of integration into eight subintervals.

   (e) Write down a left-endpoint Riemann sum that approximates the value of $\int_{-2}^{2} 7x^2 \, \mathrm{d}x$ by subdividing the domain of integration into ten subintervals.

7. (Riemann Sums, FYI) In 1994, a paper titled *A Mathematical Model for the Determination of Total Area Under Glucose Tolerance and Other Metabolic Curves* was published. Here's its abstract:

   > Objective: To develop a mathematical model for the determination of total areas under curves from various metabolic studies.
   >
   > Research Design and Methods: In Tai's Model, the total area under a curve is computed by dividing the area under the curve between two designated values on the x-axis (abscissas) into small segments (rectangles and triangles) whose areas can be accurately calculated from their respective geometrical formulas. The total sum of these individual areas thus represents the total area under the curve.

   This paper provides a nice example of calculus being used in medical research, but is also a sad instance of multiple scientists, the author all the reviewers, being unaware of a basic idea of integral calculus: that you can estimate the area under a curve by approximating that area with simpler geometric shapes like rectangles or trapezoids. So, as future scientists, I implore you to please pay attention in your math classes. For more information on this paper, see

   academia.stackexchange.com/q/9602.

8. This is kinda a trick question: Why is the following equation true? Try to notice why on your own first, but my explanation is at the end of this document.

$$\int_0^{42} \frac{\sqrt{\cos(t^2+7t)}}{\ln(23t)}\,\mathrm{d}x \quad = \quad 42\frac{\sqrt{\cos(t^2+7t)}}{\ln(23t)}$$

9. (SPIVAK, CHALLENGE) With the idea behind Exericise 8 in mind, write the following expression solely in terms of $\int_a^b f(x)\,\mathrm{d}x$ and $\int_c^d g(y)\,\mathrm{d}y$. HINT: it's crucial that you recognize what's constant relative to each integral.

$$\int_a^b \left(\int_c^d f(x)g(y)\,\mathrm{d}y \right) \mathrm{d}x$$

10. Suppose I tell you that

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{-\frac{1}{2}x^2}\,\mathrm{d}x \;=\; 1. \qquad (\star)$$

The integrand above gives you a normal (bell) distribution that is very important in statistics. Using the equality $(\star)$, what are the values of the following integrals?

$$\int_{-\infty}^{\infty} \mathrm{e}^{-\frac{1}{2}x^2}\,\mathrm{d}x \qquad\qquad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{3}\mathrm{e}^{2-\frac{1}{2}x^2}\,\mathrm{d}x$$

11. (CUTE, KINDA TRICKY) Let's evaluate the integral $\int_0^\pi \cos^2(\theta)\,\mathrm{d}\theta$ a clever way. First, look at the graphs of $\cos^2(\theta)$ and $\sin^2(\theta)$ on the interval $[0,\pi]$, and use those graphs to convince yourself that

$$\int_0^\pi \sin^2(\theta)\,\mathrm{d}\theta \quad = \quad \int_0^\pi \cos^2(\theta)\,\mathrm{d}\theta.$$

Next remember that $\sin^2(\theta) + \cos^2(\theta) = 1$. Then separately apply these two facts to this integral and play around with it a bit.

$$\int_0^\pi \left(\sin^2(\theta) + \cos^2(\theta)\right)\,\mathrm{d}\theta.$$

12. Find three *different* functions of x that each have a derivative equal to $\frac{1}{x}$. HINT: What just "goes away" when you take a derivative?

13. Find two *different* functions that have a derivative equal to $\cos(x)+2x+3$.

14. (INTEGRAL DRILLS) Using the fundamental theorem of calculus, evaluate the following integrals. Some of these are definite integrals, in which case "evaluate" means to find the *number* they are equal to. Some of these are *in*definite integrals though, in which case "evaluate" means to write down the family of antiderivatives of the function being integrated. Don't forget your $+C$!

$$\int x\,\mathrm{d}x \qquad \int_{-1}^{3} x^2\,\mathrm{d}x \qquad \int_{-2}^{2} -3t^3+\frac{1}{2}t-1\,\mathrm{d}t$$

$$\int u^9-\frac{2}{3}\,\mathrm{d}u \qquad \int (v-2)^2\,\mathrm{d}v \qquad \int_{1}^{3} x(x-1)(x+2)\,\mathrm{d}x$$

$$\int_0^{\pi/4} 5\sec^2(\theta)\,\mathrm{d}\theta \qquad \int x^{2/3}\,\mathrm{d}x \qquad \int 3\mathrm{e}^t+t+\mathrm{e}\,\mathrm{d}t$$

$$\int \mathrm{e}^{100+x}\,\mathrm{d}x \qquad \int_{\mathrm{e}}^{\mathrm{e}^2} 11\mathrm{e}^{1+t}\,\mathrm{d}t \qquad \int \mu^3\sqrt{\mu}\,\mathrm{d}\mu$$

$$\int \csc^2(\theta)+7\frac{1}{\theta}\,\mathrm{d}\theta \qquad \int_1^{\mathrm{e}} \frac{1}{t}-t\,\mathrm{d}t \qquad \int \frac{x-2}{x^2+x-6}\,\mathrm{d}x$$

$$\int_0^{\log_3(4)} 3^x\,\mathrm{d}x \qquad \int x^3+x^2+x+1\,\mathrm{d}x \qquad \int_{-1}^{1} x^3+x^2+x+1\,\mathrm{d}x$$

$$\int_1^{1} x^3+x^2+x+1\,\mathrm{d}x \qquad \int_1^{8} t^{-2/3}\,\mathrm{d}t \qquad \int \frac{\mathrm{d}x}{\sin^2(x)-1}$$

$$\int \frac{2x^2-x}{\sqrt{x}}\,\mathrm{d}x \qquad \int \frac{x^2}{x^2+1}\,\mathrm{d}x \qquad \int_{-1}^{0} 4x\,\mathrm{d}t$$

$$\int \cos(2\theta)\,\mathrm{d}\theta \qquad \int \mathrm{e}^{2t}\,\mathrm{d}t$$

If you want more practice at this sort of exercise you can look at the end of section 6.2 of Neuhauser. But we will be evaluating *many* more integrals as the class progresses.

15. (I think this sort of exercise is silly, but it's classic, so I guess I have to give you some practice.) Using the first part of the fundamental theorem of calculus, take the following derivatives.

$$\frac{d}{dx}\int_3^{x^3} \ln(u+u^2)\,du \qquad \frac{d}{dt}\int_{\sin(t)}^{0} \sin(x)\,dx$$

$$\frac{d}{dx}\int_{\ln(x)}^{\tan(x)} w^7\,dw \qquad \frac{d}{dx}\int_3^{7} 3e^t\,dt$$

HINT: Remember it's the chain rule. If you want more practice at this sort of exercise you can look at the end of section 6.2 of Neuhauser.

16. For each of the following, find the (unsigned) area of the region in the (x, y)-plane bounded by the listed curves. Be careful, because the region that gets enclosed by the curves might not be a *connected* region (there might be more than one chunk).

(a) $y = x^2 - 4$ and $y = x + 2$

(b) One of the chunks bounded by $y = \cos(x)$ and $y = 1$

(c) $y = \sec^2(x)$, $y = \cos(x)$, $x = 0$, and $x = \pi/4$

(d) $y = \frac{1}{x}$, and $y = 5 - x$

(e) $y = x^2$, $y = x^3$ and $x = 3$

(f) $y = x^2$ and $y = x^4$

(g) $y = e^x$, $x = -3$, and $y = 4$

If you want more practice at this sort of exercise you can look at the end of section 6.3 of Neuhauser.

My Solutions to Exercise 2

12	10	2π
0	$-\frac{41}{6}$	30

My Solutions to Exercise 4

They're all zero.

My Solutions to Exercise 6

18	-5
-28	$\frac{1}{4}\left(\frac{1}{2}^3+1^3+\frac{3}{2}^3+\cdots+4^3\right)$
$7\cdot\frac{2}{5}\left(-2^3-\frac{8}{5}^3-\frac{6}{5}^3-\cdots+\frac{8}{5}^2\right)$	

My Explanation of Exercise 8

We're taking the integral *with respect to x*! That means x is our only variable here. What even is t? Relative to x, t is just some constant, and any expression involving t is just some constant. So that entire expression involving t pulls out front of the integral.

My Solutions to Exercise 10

$\sqrt{2\pi}$	$3e^{-2}$

My Solutions to Exercise 12

Any function of the form $f(x) = \ln(x) + C$ for a real number C will do.

My Solutions to Exercise 15

$3x^2\ln(x^3+x^6)$	$-\sin(\sin(t))\cos(t)$
$\tan^7(x)\sec^2(x)-\frac{1}{x}\big(\ln(x)\big)^7$	0 (you're taking the derivative of a constant)