Homework Two

Integral Calculus for Life Sciences
UCR Math-007B-B01, Summer 2019

1. (NEUHAUSER) A particle moves along the x-axis with velocity given by by the function $v(t) = -(t-2)^2 + 1$ for $t \in [0,5]$. Declare the location of the particle at time zero to be the origin.

   (a) Graph the function $v(t)$, and use the graph to determine when the particle moves to the right and when the particle moves to the left.

   (b) Write a function for the location of the particle for times $t \in [0,5]$.

   (c) Find the left-most and right-most positions that the particle reaches.

2. If $\ell'(t)$ represents the growth rate of a worm at a time t months into its life, what does this definite integral represent?

$$\int_3^5 \ell'(t)\,\mathrm{d}t$$

3. (NEUHAUSER) Suppose that the temperature T of a greenhouse, measured in degrees Fahrenheit, varies over a 24 hour period according to the function

$$T(t) = 68 + \sin\left(\frac{\pi}{12}t\right)$$

   for t measured in hours. Find the average temperature of the greenhouse in that 24 hour period. Then graph the function $T(t)$ and explain why that average temperature makes sense based on the geometry of the graph.

4. (MICROTUTORIAL) The population *density* of a plant species living on the banks of a river is given by the function $f(x)$ in individuals per square meter, where x is the distance from a river. There are no plants further than 100 ft from the river, so $f(x) = 0$ for $x \geq 100$. Write down a definite integral to calculate the number of plants along a section of the river of length L. To see an example similar to this one worked through, watch

   blendedmathematics.ucr.edu/plant_density.html

5. (MICROTUTORIAL) The infection level of a of the measles virus in a particular patient has been modelled by the function $f(t) = -t(t-21)(t+1)$ where t is measured in days, and $f(t)$ (the infection level) is measured in the number of infected cells per mL of blood plasma. Over the course of the 21 day infection, what is the average level of infection? To see an example similar to this one worked through, watch

   blendedmathematics.ucr.edu/length_of_a_fish.html

6. (MICROTUTORIAL) You're studying a population of trout in a 20 mile section of stream. The population density of trout in this section of stream is given by the function
$$\rho(x) = |-x^2 + 5x + 50|$$
   where $\rho(x)$ is measured in trout per mile and x is measured in miles running from 0 to 20 miles. Graph the function ρ and use the graph to find the places along the stream where the density of trout is minimal and maximal. What is the total number of trout in the stream? What is the average density of trout along the 20 mile stretch of stream? Based on your graph, find a location along the stream where the trout population density is about the same as the *average* trout population density along the entire 20 mile stretch of stream.

   blendedmathematics.ucr.edu/stream_of_trout.html

7. You are standing on the shoulder of a straight freeway somewhere in Nevada. There is a Porsche 911 Carrera S stopped on the freeway far to your left. You start your stopwatch, and the Porsche floors it and heads towards you with a constant acceleration of 54 000 miles/hr^2. The car races past you exactly 30 seconds after it starts. How far away from you was the car initially? Determine how fast was the car going when it passed you, and realize that it's unrealistic to model a car's acceleration with a constant function. [1] What function do you think would better model the acceleration of the Porsche?

[1]According to Consumer Reports, this is roughly accurate, but only for the first 4.1 seconds that the car is moving. Among the cars they tested the Porsche 911 Carrera S had the best average acceleration, doing 0 mph to 60 mph in 4.1 seconds. So the average acceleration is
$$\frac{60}{4.1} \approx 15 \frac{\text{miles}}{\text{hours} \times \text{seconds}}.$$
But the units on this are dumb. Doing some brisk unit conversion, this is equivalent to $22\,\frac{\text{ft}}{\text{s}^2}$, or $6.7\,\frac{\text{m}}{\text{s}^2}$, or $54\,000\,\frac{\text{miles}}{\text{hr}^2}$. This is pretty good. The average car gets between $3\,\frac{\text{m}}{\text{s}^2}$ and $4\,\frac{\text{m}}{\text{s}^2}$.

8. (MICROTUTORIAL) Suppose the volume of a cell is increasing at a constant rate of $10^{-6}\,\text{cm}^3/\text{s}$. Write down a function $V(t)$ that models the volume of the cell at time t, given that the volume of the cell at time $t = 0$ seconds is 13×10^{-5}. What is the volume of the cell at $t = 10$ seconds? To see an example similar to this one worked through, watch

   blendedmathematics.ucr.edu/population_dynamics.html

9. (ARCLENGTH) Write down an integral that represents the arclength of the given curves between the indicated points. The integrals you get will almost certainly be tough to evaluate with what we know now, so there's no need to try to evaluate them yet.

   (a) The curve given by $y = \sin(x)$ between the points $(0,0)$ and $(\pi/2, 1)$.

   (b) The curve given by $y = \frac{1}{3}x^3 + x^2 + 7$ between the points $(0,7)$ and $(3,25)$.

   (c) The curve given by $y = \ln(\sec(\theta))$ between the points $(0,0)$ and $\left(\frac{\pi}{4}, \ln(\sqrt{2})\right)$.

10. (NEUHAUSER, ARCLENGTH) Suppose you have two poles that are posted at $x = -M$ and $x = M$, and between the two poles you hang a cable. The cable will droop under the force of gravity, with it's lowest point being over $x = 0$. The shape that this cable makes is called a catenary, and is modelled by the graph of the equation
$$y = \frac{1}{2a}\left(e^{ax} + e^{-ax}\right),$$
where the constant a depends on any slack is in the cable and the force of gravity. Compute the length of such a cable if $a = 1$ and $M = \ln(2)$.

11. (GEOMETRY) For each of the following, using whatever method you like, find the volume of the solid that results from rotating the indicated region about the indicated axis. Drawing a picture of the solid being described will help you set up the correct integral.

   (a) The region bounded by the curves $y = 4 - x^2$, $y = 0$, and $x = 0$, rotated about the x-axis.

   (b) The region bounded by $y = \sqrt{\sin(x)}$ and the x-axis between $x = 0$ and $x = \frac{\pi}{2}$, rotated about the x-axis.

   (c) The region bounded by $y = 2 - x^3$ and $y = 2 + x^3$ for x between 0 and 1, rotated about the x-axis.

(d) The region bounded between $y = \ln(x+1)$, $y = 3$, and $x = 0$, rotated about the y-axis.

(e) The region bounded between $y = \mathrm{e}^x$, $y = \mathrm{e}^{-x}$, $x = 0$ and $x = 2$, rotated about the x-axis.

(f) The region bounded between $y = \frac{1}{x}$, $x = 0$, $y = 1$, and $y = 2$, rotated about the y-axis.

(g) The region bounded between $y = \frac{1}{x}$, $x = 0$, $y = 1$, and $y = 2$, rotated about the x-axis.

If you want more practice at this sort of exercise you can look at the end of section 6.3 of Neuhauser.

12. (GEOMETRY) Calculate the following volumes. Drawing a picture of the solid being described will help you set up the correct integral.

    (a) Find the volume of a sphere of radius 1 by taking the half-circular region of radius 1 that lives below the curve $y = \sqrt{1-x^2}$ and rotating it about the x-axis. Then alter this calculation to find the general formula for the volume of a sphere of radius r.

    (b) Determine a formula for the volume of a right-circular cone (a cone with a circular base such that that the "point" of the cone is directly over the center of the base) with height h and circular base of radius r.

    (c) Determine a formula for the volume of a square-based pyramid with height h and with a base of side-length ℓ. Drawing a picture of the solid being described will help you set up the correct integral.

13. (SPIVAK) Imagine a solid that has circular base with diameter $\overline{AB}$ with length ℓ such that each plane that is perpendicular to $\overline{AB}$ intersects the solid in a square. Express the volume of this solid as an integral and then evaluate the integral.

14. (CHALLENGE, PROFESSOR GAN) You have a bowl full of water, the shape of which is exactly half of a sphere of radius r. You tilt the bowl thirty degrees, spilling out some of the water. What is the volume of the remaining water?