IntBioCalc Week 4 Monday

Integrals with lots of trig functions.

Let's start with an easy one.

$$\int \cos(x)\sin^5(x)\,dx$$ PAUL

This is accomplished with a quick substitution

Let $u = \sin(x)$, so $du = \cos(x)\,dx$

$$\int \cos(x)\sin^5(x)\,dx = \int u^5\,du = \frac{1}{6}u^6 + C = \frac{1}{6}\sin^6(x) + C$$

It was a good thing we had that cosine there, or else our substitution wouldn't have worked.

Sometimes though we've gotta <u>make</u> our substitution work. It'll help to remember

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

$$\int \sin^5(x)\,dx$$

We only have a bunch of sines, so letting $u = \sin(x)$ won't work since there is no cosines.

The idea is to make all the sines into cosines ~~All~~ EXCEPT FOR ONE,

$$\int \sin^5(x)\,dx = \int \sin(x)\left(1-\cos^2(x)\right)^2 dx$$

$$= \int \sin(x)\left(1-2\cos^2(x)+\cos^4(x)\right) dx$$

$$= \int \sin(x)\,dx - 2\int \sin(x)\cos^2(x)\,dx + \int \sin(x)\cos^4(x)\,dx$$

Now we can do a sustitution to solve these.

Let $u = \cos x$, so $du = -\sin(x)\,dx$, and

$$= \int \sin(x) + 2\int u^2\,du - \int u^4\,du$$

$$= -\cos(x) + \frac{2}{3}\left(\cos(x)\right)^3 - \frac{1}{5}\left(\cos(x)\right)^5 + C$$

This trick will totally work if your integral is ~~one~~ of the form $\int \sin^a(x)\cos^b(x)\,dx$ if one of a or b is odd, because that gives you 1 copy left over to be your du after the substitution. But what if both a and b are even?

$$\int \cos^2\theta \, d\theta$$

Then you're gotta sub everything using the half/double angle formulas:

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2} \qquad \sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\int \cos^2\theta\, d\theta = \int \frac{1+\cos(2\theta)}{2}\, d\theta = \frac{1}{2}\int 1\, d\theta + \frac{1}{2}\int \cos(2\theta)\, d\theta = \ldots$$

$u = 2\theta$
$\frac{1}{2}du = d\theta$

$$\ldots = \frac{1}{2}\theta + \frac{1}{4}\int \cos(u)\, du$$

$$\ldots = \frac{1}{2}\theta + \frac{1}{4}\sin(u) + C$$

$$= \frac{1}{2}\theta + \frac{1}{4}\sin(2\theta) + C$$

Now you can do this same idea with $\sec\theta$ and $\tan\theta$ in your integrands. Remember

$\tan^2\theta + 1 = \sec^2\theta$ $\quad \frac{d}{dx}\tan(x) = \sec^2 \cancel{x}$ $\quad \frac{\partial}{\partial x}\sec(x) = \sec(x)\tan(x)$

$$\int \sec^9(x)\tan^5(x)\,dx$$

Let's ~~chan~~ put aside a single ~~sec~~ $\sec(x)\tan(x)$ and change all the tangents to secants to use the substitution $\underline{u = \sec}(x)$, so $\underline{du = \sec(x)\tan(x)}\,dx$

$$\int \sec^9(x)\tan^5(x)\,dx = \int \sec^8(x)\tan^4(x)\left(\sec(x)\tan(x)\,dx\right)$$

$$= \int \sec^8(x)\Big(\overset{\sec^2(x)}{\cancel{\tan^2(x)}}+1\Big)^2\left(\sec(x)\tan(x)\,dx\right)$$

$$\sec^4(x)+2\sec^2(x)+1$$

$$\cancel{= \int \sec^{12}(x)\left(\sec(x)\tan(x)\right)dx}$$

$$\cancel{= \int u^{12}\,du = \frac{1}{13}u^{13} + C}$$

$$\cancel{= \frac{1}{13}}$$

$$\int \sec^8(x)\left(\sec^4(x)+2\sec^2(x)+1\right)\left(\sec(x)\tan(x)\,dx\right) = \ldots$$

$$= \int u^8 (u^4 + 2u^2 + 1)\, du$$

$$= \int u^{12}\, du + 2\int u^{10}\, du + \int u^8\, du$$

$$= \frac{1}{13}\sec^{13}(x) + \frac{2}{11}\sec^{11}(x) + \frac{1}{9}\sec^{9}(x) + C$$

We could only do that because there was an odd power on tangent (to set aside a tangent). If we had an even power on secant we can do the same sort of trick by putting aside a $\sec^2$ and converting the rest to tangent

$$\int \sec^{10}(x)\tan^2(x)\, dx$$

$$= \int \sec^8(x)\tan^2(x)\,(\sec^2(x)\, dx)$$

$$= \int (\tan^2(x)+1)^4 \tan^2(x)\,(\sec^2(x)\, dx)$$ Let $u = \tan(x)$, $du = \sec^2(x)$

$$= \int (u^2+1)^4 u^2\, du = \ldots \text{ doable}$$

If $\int \sec^{odd}(x) \tan^{even}(x)\, dx$, life might be tougher, and you have to deal with it differently: either convert EVERYTHING to ~~the~~ secant and start digging, or maybe convert everything to sines and cosines and try your luck there.

(2) $\int \sin^2\theta \cos^2(\theta)\, d\theta$

(1) $\int \sin^5(x) \cos^4(x)\, dx$

(3) $\int \tan^4(x) \sin^3(x)\, dx$

$\left(= \int \frac{\sin^7(x)}{\cos^4(x)}\, dx = \int \frac{(1-\cos^2 x)^3}{\cos^4(x)} \sin(x)\, dx \right)$

(4) $\int \sec^4(x) \tan(x)\, dx$

You can do analagous stuff for $\csc(x)$ $\cot(x)$ functions

Int Bio Calc Week 4 ~~Monday~~ Tuesday

Trig - Substitution

Today we're going to talk about a different kind of substitution. Mechanically it's the same as the usual sort of substitution, but the motivation is different. Instead of letting $u =$ gross-stuff with x we're going to take x and send it to a trigonometric thing so that we can use some trig identities, in particular

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta.$$

Each of these has the form of a square equalling $\pm$ a square ± 1, which will be useful. For example, consider the following integral.

$$\int \frac{1}{x^2+1}\,dx$$

We "know" it's equal to $\arctan(x) + C$, but now I'll show you how to calculate it.

We can't just let $u = x^2+1$, because the $du = 2x\,dx$ has an x that we can't find in the integrand.

But the thing that makes this tough is that sum in the denominator. We wish that was a single term.

Now recall that $\underbrace{\tan^2\theta + 1}_{x^2\,+\,1} = \sec^2\theta$. Wouldn't it be nice if that x were a $\tan(\theta)$ instead to get rid of the sum on the bottom!? We can do that.

Let $x = \tan\theta$, so $dx = \sec^2\theta\, d\theta$.

$\Rightarrow \arctan(x) = \theta$

$$\int \frac{1}{x^2+1}\,dx = \int \frac{1}{(\tan\theta)^2+1}\left(\sec^2\theta\, d\theta\right)$$

$$= \int \frac{\sec^2\theta}{\tan^2\theta+1}\,d\theta = \int \frac{\sec^2\theta}{\sec^2\theta}\,d\theta = \int 1\,d\theta$$

$$= \theta + C = \arctan(x) + C \,/\!/$$

That wasn't so bad! And we can try this trick anytime we have a (±square ± 1) we don't like, and use the trig identity of those two that fit. Remember we can rearrange those identities

$$\begin{cases} 1 - \cancel{\cos^2\theta}^{\,\sin^2\theta} = \cancel{\sin^2\theta}^{\,\cos^2\theta} \\ 1 + \tan^2\theta = \sec^2\theta \\ \sec^2\theta - 1 = \tan^2\theta . \end{cases}$$

What substitutions should we make for these following integrals?

$$\int \sqrt{1-x^2}\,dx \qquad \int \frac{1}{\sqrt{x^2-1}}\,dx \qquad \int \frac{x}{\sqrt{x^2-1}}\,dx$$

$x \mapsto \sin\theta$ ... $\int \cos^2\theta \, d\theta$ second

$x \mapsto \sec\theta$ ... $\int \sec\theta \, d\theta$ first

u-sub! $u = x^2 - 1$!

Let's go ahead and work through that second one.

$$\int \frac{1}{\sqrt{x^2-1}}\,dx$$

Let $x = \sec\theta$ so $dx = \sec\theta\tan\theta\,d\theta$, and

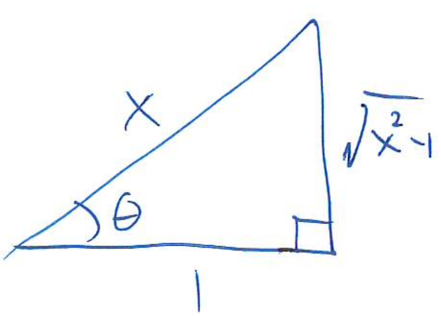
$$\int \frac{1}{\sqrt{x^2-1}}\,dx = \int \frac{1}{\sqrt{\sec^2\theta-1}}(\sec\theta\tan\theta)\,d\theta = \int \frac{\sec\theta\cancel{\tan\theta}}{\cancel{\tan\theta}}\,d\theta$$

$$= \int \sec\theta\,d\theta \overset{\text{remember}}{=} \ln(\sec\theta + \tan\theta) + C = \dots$$

But $x = \sec\theta$, so $\operatorname{arcsec} x = \theta$.

$$\therefore = \ln\left(\sec(\operatorname{arcsec} x) + \tan(\overbrace{\operatorname{arcsec}(x)}^{\theta})\right) + C$$

$$= \ln\left(x + \sqrt{x^2-1}\right) + C \; //$$



Now these have been of a pretty nice form, but what if I give you something uglier?

$$\int \frac{dx}{x^2+2x+3}$$

Since x^2+2x+3 can't factor further, we can't do partial fraction stuff. If you could force it into the form ($\pm$square ± 1) that would be great. We can!

$$x^2+2x+3 = x^2+2x+1-1+3$$

$$= \cancel{x} \ (x+1)^2+2 = 2\left(\left(\frac{x+1}{\sqrt{2}}\right)^2+1\right)$$

Let $\frac{x+1}{\sqrt{2}} = \tan\theta$, so $\frac{1}{\sqrt{2}}dx = \sec^2\theta \, d\theta$, and

$$\int \frac{1}{x^2+2x+3}dx = \int \frac{dx}{2\left(\left(\frac{x+1}{\sqrt{2}}\right)^2+1\right)} = \frac{1}{2}\int \frac{\sqrt{2}\sec^2\theta \, d\theta}{\tan^2\theta+1}$$

$$= \frac{1}{2}\int \sqrt{2}\, d\theta = \frac{\sqrt{2}}{2}\theta = \frac{\sqrt{2}}{2}\arctan\left(\frac{x+1}{\sqrt{2}}\right)+C \ //$$

$$\int \frac{\sqrt{25x^2-4}}{x}\,dx$$

Now what if I give you an integral like this?
You still need to put whats under the radical in the right form:

$$25x^2-4 = 4\left(\frac{25}{4}x^2-1\right) = 4\left(\left(\frac{5}{2}x\right)^2-1\right)$$

So Let $\frac{5}{2}x = \sec\theta$, so $\frac{5}{2}dx = \sec\theta\tan\theta\,d\theta$,

$x = \frac{2}{5}\sec\theta$

$$\int \frac{\sqrt{25x^2-4}}{x} = \int \frac{\sqrt{4((\sec\theta)^2-1)}}{\cancel{\frac{2}{5}}\cancel{\sec}\theta}\,\cancel{\frac{2}{5}}\cancel{\sec}\theta\tan\theta\,d\theta$$

$$= 2\int \tan^2\theta\,d\theta = 2\int \sec^2\theta\,d\theta - 2\int d\theta = \ldots$$

$$\ldots = 2\tan\theta - 2\theta + C = 2\tan\left(\operatorname{arcsec}\left(\frac{5x}{2}\right)\right) - 2\operatorname{arcsec}\left(\frac{5x}{2}\right) + C$$

$$= \cancel{2}\left(\frac{\sqrt{25x^2-4}}{\cancel{2}}\right) - 2\operatorname{arcsec}\left(\frac{5x}{2}\right) + C$$

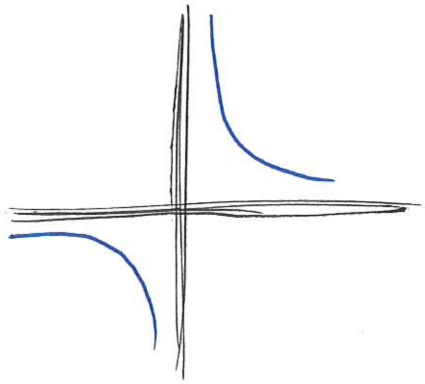
Practice

$$\int \frac{1}{x^2-1}\,dx$$

$$\int \sqrt{1-7w^2}$$

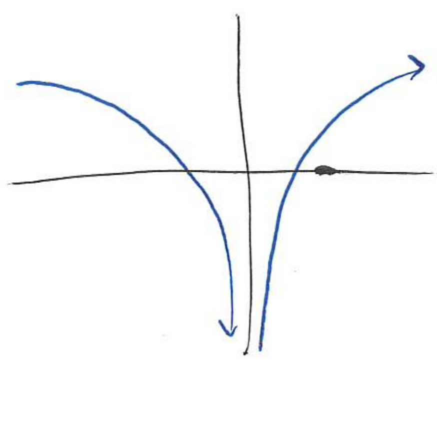
$$\int e^{4x}\sqrt{1+e^{2x}}\,dx$$

I thought we'd get through this class without having to think about domain restrictions. I was ~~was~~ wrong. Consider the function $f(x) = \frac{1}{x}$.



It's defined everywhere besides $x=0$. Now ~~we~~ ~~[illegible]~~ since $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$, we figured that $\int \frac{1}{x}\,dx = \ln(x) + C$, but alas! $\ln(x)$ is only defined on positive numbers! This doesn't make sense. It should, as an antiderivative of $\frac{1}{x}$ be defined everywhere $\frac{1}{x}$ is. So, honestly,

$$\int \frac{1}{x}\,dx = \ln(|x|) + C$$
$$= \ln|x| + C$$



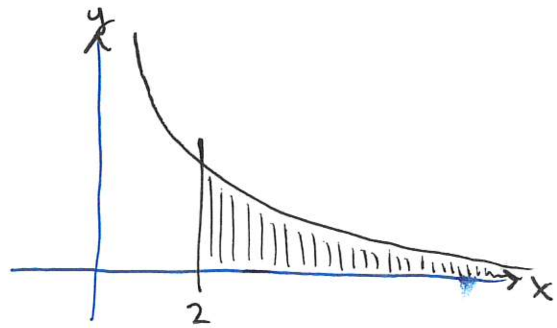
Addendum!

Int BioCalc Week 4 Wednesday

"Improper Integrals"

One example of an improper integral is a definite integral where the bounds of integration tend to infinity. For example

$$\int_2^\infty \frac{1}{x^2}\,dx$$



This notation is just shorthand for

$$\lim_{R\to\infty} \int_2^R \frac{1}{x^2}\,dx$$

And even though the bound is tending to infinity, the area being ~~[illegible]~~ swept out might still be finite... or it might not too.

$$\int_2^\infty \frac{1}{x^2}\,dx = \lim_{R\to\infty} \int_2^{R} \frac{1}{x^2}\,dx = \lim_{R\to\infty}\left(-\frac{1}{x}\Big|_2^R\right) = \cdots$$

$$\cdots = \lim_{R\to\infty}\left(-\frac{1}{R} - \left(-\frac{1}{2}\right)\right)$$

$$= -\lim_{R\to\infty}\frac{1}{R} + \frac{1}{2} = 0 + \frac{1}{2} = \frac{1}{2}$$

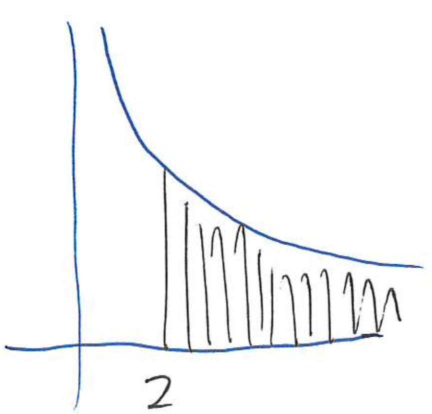
Contrast that with this example:

$$\int_2^\infty \frac{1}{\sqrt{x}}\,dx$$

$$\cdots = \lim_{R\to\infty}\int_2^R \frac{1}{\sqrt{x}}\,dx = \lim_{R\to\infty} 2\sqrt{x}\Big|_2^R$$

$$= 2\lim_{R\to\infty}\sqrt{R} - 2\sqrt{2}$$

Grows infinitely large.



Though the graphs look about the same, this one doesn't "decrease fast enough" and so will sweep out infinitely much area.

In this first case when the area is finite we'll say the integral <u>converges</u>. Otherwise, like in this last case, we'll say the integral <u>diverges</u>. Now we can have infinite bounds on the left, or even on both sides too:

$$\int_{-\infty}^{0} f(x)\, dx = \lim_{L \to -\infty} \int_{L}^{0} f(x)\, dx$$

$$\int_{-\infty}^{\infty} f(x)\, dx = \lim_{L \to -\infty} \int_{L}^{0} f(x)\, dx + \lim_{R \to \infty} \int_{0}^{R} f(x)\, dx$$

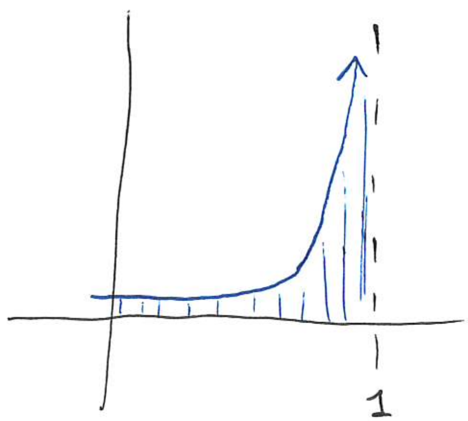
And these are handled similarly.

Now there is another sort of "improper integral," ~~when the unbounded region area occurs in the bounds of integration. Like~~ When the function is not defined, and admits an asymptote, at some spot in ~~the~~ between the bounds of integration.

For example

$$\int_0^1 \frac{1}{(x-1)^{2/3}}\,dx$$

The integrand has an asymptote at $x=1$.



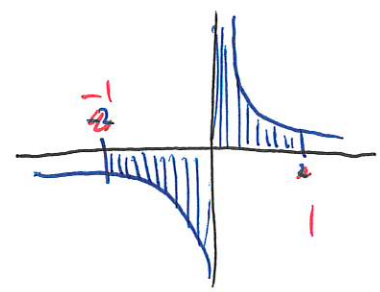
Similar to the other case, we have to evaluate this integral by taking a limit: the antiderivite of $\frac{1}{(x-1)^{2/3}}$ won't be defined at $x=1$ either, so we can't simply plug that bound in.

$$\int_0^1 \frac{1}{(x-1)^{2/3}}\,dx = \lim_{r\to 1^-} \int_0^r \frac{1}{(x-1)^{2/3}}\,dx$$

But dissimilarly to the other type of improper integral, we aren't told ~~that the integ~~ that it's improper: we have to notice ourselves that the integrand has a discontinuity on the bounds of integration and consider a limit.

That last example had a discontinuity AT one of the bounds. But a discontinuity could very well be in the middle of the interval along with we're integrating:

$$\int_{-1}^{1} \frac{1}{x}\,dx$$



For these we have to break them up into two separate integrals with limits

$$\int_{-1}^{1} \frac{1}{x}\,dx = \lim_{r \to 0^-} \int_{-1}^{r} \frac{1}{x}\,dx + \lim_{\ell \to 0^+} \int_{\ell}^{1} \frac{1}{x}\,dx$$

You might be tempted to say that the value of that integral $\int_{-2}^{2} \frac{1}{x}\,dx$ must be zero since $f(x) = \frac{1}{x}$ is an odd function, and the areas cancel out, but infinity is fickle. IF THE INTEGRAL CONVERGES, then it's value will be zero, but it might not converge at all. After all, those two limits/integrals

$$\lim_{r \to 0^-} \int_{-1}^{r} \frac{1}{x}\,dx + \lim_{l \to 0^+} \int_{l}^{1} \frac{1}{x}\,dx$$

can't cancel each other out if they each don't have a value. In fact, looking at the left one,

$$\lim_{r \to 0^-} \int_{-1}^{r} \frac{1}{x}\,dx = \lim_{r \to 0^-} \left(\ln(|x|) \Big|_{-1}^{r} \right)$$

$$= \lim_{r \to 0^-} \ln|r| - \ln|-1| = .$$

Does not converge

Rather famously, that area $\int_0^1 \frac{1}{x}\,dx$ is unbounded.

And notice that you CAN'T FORGET that you must break up an integral at a discontinuity like that. Otherwise you'll get a numerical answer when an integral actually diverges. ☹

Now let's practice.

$$\int_0^\infty 3e^{-6x}\,dx$$

Conv.

$$\int_{-1}^{1} \frac{1}{x^2}\,dx$$

Div

$$\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^2}\,dx$$

conv.

$$\int_1^e \frac{dx}{x\ln(x)}$$

?

Int Bio Calc Week 4 Thursday

Today, let's do a bit of an INTEGRATION ~~Blue~~. MARATHON. I'll keep em' coming :) .

Let's start with some ~~straightforward~~ ones from this week.

$\int 4xe^{-x^2}\,dx$ $\underline{u\text{-sub}}$ $u=-x^2$

$\int x\sec^2(3x^2)\,dx$ sub $u=3x^2$ $\int \sec^2\theta = \tan\theta + c$

$\int \underset{\partial v}{x}\,\underset{u}{\ln(x)}\,dx$ parts $\partial v = x\,dx$ $v=\frac{1}{2}x^2\,dx$ $u=\ln(x)$ $\partial u = \frac{1}{x}\,dx$

$= \frac{1}{2}x^2\ln(x) - \int \frac{1}{2}x\,dx = x^2\ln(\sqrt{x}) - \frac{1}{4}x^2 + C$ //

$\int \tan\theta\,\sec^2\theta\,\partial\theta$ sub $u=\tan\theta$ ✓

$\int \sqrt{e^x}\,dx$ tough sub $u=e^x$ $\partial u = e^x\,dx = u\,dx$ $= \int \sqrt{u}\,\frac{1}{u}\,\partial u = \int u^{-1/2}\,\partial u = \dots$ ✓

$$\int \sin(x)\cos(x)\, e^{\sin(x)}\, dx \qquad \begin{aligned} u &= \sin(x) \\ du &= \cos(x)\, dx \end{aligned}$$

sub then parts

~~partial~~ partial fracs

$$\int \frac{1}{(x+1)(x-2)}\, dx = \int \frac{A}{x+1} + \frac{B}{x-2}\, dx = -\frac{1}{3}\ln(x+1) + \frac{1}{3}\ln(x-2) + C$$

$$\Rightarrow \quad Ax - 2A + Bx + B = 1 \quad \Rightarrow \quad \begin{cases} A+B=0 & A=-B \\ -2A+B=1 \end{cases}$$

$$2B+B=1 \qquad B=\frac{1}{3} \quad A=-\frac{1}{3}$$

* $\displaystyle\int \frac{2x+1}{\sqrt{1-x^2}}\, dx$ sub, but also $\overset{\text{easy}}{\vee}$ trig sub :)

$u = 1-x^2$

$du = -2x\, dx$

$x \mapsto \sin\theta$

$dx \mapsto \cos\theta\, d\theta$

$$= \int \frac{2x}{\sqrt{1-x^2}}\, dx + \int \frac{1}{\sqrt{1-x^2}}\, dx = -\int \frac{du}{\sqrt{u}} + \int \frac{\cos\theta}{\cos\theta}\, d\theta$$

* $\displaystyle\int_0^2 \frac{1}{4+x^2}\, dx$ trig sub (good for final)

$\arctan\left(\frac{x}{2}\right) = \theta$

$\frac{x}{2} \mapsto \tan\theta$

$\frac{1}{2}dx \mapsto \sec^2\theta$

$$= \int_0^2 \frac{1}{4\left(1+\left(\frac{x}{2}\right)^2\right)}\, dx = \int_{\underset{0}{\arctan 0}}^{\overset{\pi/4}{\arctan(1)}} \frac{1}{4(\cancel{\sec^2\theta})}(\cancel{\sec^2\theta})\, d\theta = \frac{1}{4}\int_0^{\pi/4} 1\, d\theta \ldots$$

$\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\, dx$ improper :)

$$= \lim_{\ell \to 0^+} \int_\ell^1 \frac{1}{\sqrt{x}}\, dx = \lim_{\ell \to 0^+} \left(2\sqrt{x}\,\Big|_\ell^1\right) = 2 - 2\lim_{\ell \to 0^+} \sqrt{\ell}$$

$$= 2 \,/\!/$$

$$\int_1^\infty \frac{1}{\sqrt{x}}\,dx$$ improper

$$= \lim_{R\to\infty} \int_1^R \frac{1}{\sqrt{x}}\,dx = 2\lim_{R\to\infty} \sqrt{x} - 2$$

Diverges :

$$\int \frac{(x+1)^2}{x-1}\,dx \qquad \begin{aligned} u &= x-1 \\ du &= dx \end{aligned}$$

Sub, then crank it out ✓

$$\star\quad 2\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\,dx \qquad \begin{aligned} x &= \sin\theta \\ dx &= \cos\theta\, d\theta \end{aligned} \qquad 2\int_0^{\arcsin(\frac{1}{2})} \frac{1}{\cos\theta}\cos\theta\, d\theta = 2(\theta)\Big|_0^{\arcsin(\frac{1}{2})}$$

chill trig sub

$$= 2\arcsin\left(\tfrac{1}{2}\right) = \frac{\pi}{3}$$

All of these are from Neuhauser. Let's get a little messier :)

Spivak ★

$$\int \sqrt{\frac{x-1}{x+1}}\,\frac{1}{x^2}\,dx \qquad \begin{aligned} u &= x+1 \\ du &= dx \end{aligned}$$

Let $\frac{2}{u} = \sin^2\theta \Rightarrow \frac{2}{\sin^2\theta} = u$ ~~$2\cos^2\theta$~~

$$-\frac{\cancel{2}}{u^2}\,du = \cancel{2}\sin\theta\cos\theta\, d\theta$$

$$du = -u^2 \sin\theta\cos\theta$$

$$= \int \sqrt{\frac{u-2}{u}}\,\frac{1}{(u-1)^2}\,du = \int \sqrt{1-\frac{2}{u}}\,\frac{1}{(u-1)^2}\,du = \int \cos\theta \frac{1}{\left(\frac{2}{\sin^2\theta}-1\right)^2}\left(-\frac{4}{\sin^4\theta}\sin\theta\cos\theta\right)d\theta$$

$$= 4\int \frac{\cos^2\theta}{\sin^3\theta}\left(\frac{\sin^{\cancel{2}\,4}\theta}{(2-\sin^2\theta)^2}\right)$$

Must it really be that tough?

$$\int \sqrt{\tan\theta}\,d\theta \qquad \int_0^{\pi/2} \ln(\cos(t))\,dt \qquad \int \ln\left(\mu + \sqrt{1-\mu}\right)d\mu \qquad \int_0^1 \frac{\ln(x+1)}{1+x^2}\,dx$$

Putnam 2005

$$\int_0^1 \frac{\ln(x+1)}{1+x^2}\,dx$$

obvious move first

$x \mapsto \tan\theta$

$dx \mapsto \sec^2\theta\, d\theta$

$u = \tan\theta + 1$

$du = \sec^2\theta\, d\theta$

$u = \ln(x+1)$ $\quad dv = \ldots$

$du = \frac{1}{x+1}$ $\quad v = \arctan(x)$

$$\int_0^{\pi/4} \frac{\ln(\tan\theta+1)}{\sec^2\theta}\cancel{\sec^2\theta}\,d\theta = \int_0^{\pi/4} \ln(\tan\theta+1)\,d\theta = \int_0^2 \text{Booo...}$$

$u = \frac{1}{x+1}$ $\quad dv = \arctan(x)$

$du = -\frac{1}{(x+1)^2}$ $\quad v = \frac{1}{x^2+1}\,dx$

$$\int_0^1 \underbrace{\frac{1}{1+x^2}}_{dv} \underbrace{\ln(x+1)}_{u}\,dx = \ln(x+1)\arctan(x)\Big|_0^1 - \int_0^1 \frac{\arctan(x)}{x+1}\,dx$$

$$= \ln(2)\frac{\pi}{4} - \int_0^1 \frac{1}{x+1}\arctan(x)\,dx$$

$$= \ln(2)\frac{\pi}{4} - \left(\frac{1}{(x+1)(x^2+1)}\Big|_0^1 + \int \frac{1}{(x+1)^2(x^2+1)}\,dx\right)$$

$$= \frac{\pi}{4}\ln(2) - \left(\left(\frac{1}{4} - 1\right) + \int \frac{1}{(x+1)(x+1)(x^2+1)}\,dx\right)$$

And this is do-able :)

$$\int \sqrt{\tan\theta}\,d\theta$$