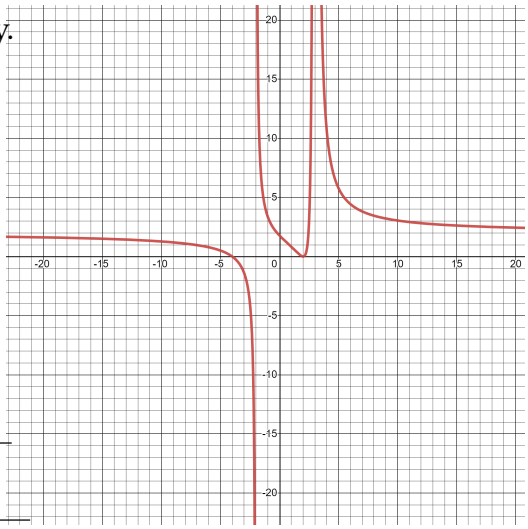
WORKSHEET 4: 18 points

Math 6B-030, Spring 2021

Due: Friday, April 30th, 11:59pm via Gradescope

Question 1 (4 points) Describe vertical asymptotes and horizontal asymptotes as if you were trying to teach a peer who enrolled in 6B late and has never learned about asymptotes before. Include in your description visual, symbolic, numeric, and verbal explanations. Give a few examples of functions that have vertical and/or horizontal asymptotes.

Question 2 (10 points) Let $r(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials. Suppose that the following is the graph of $r(x)$. Assume any zeros, holes, and asymptotes are visible and occur at integers. In the following questions, list any asymptotes correctly as lines. *No partial credit on (a)-(f).*



(a). (1 point) List ALL zeros of p(x) of EVEN multiplicity. Explain completely how you can tell.

(b). (1 point) List ALL zeros of p(x) of ODD multiplicity. Explain completely how you can tell.

(c). (1 point) List ALL zeros of q(x) of EVEN multiplicity. Explain completely how you can tell..

(d). (1 point) List ALL zeros of q(x) of ODD multiplicity. Explain completely how you can tell.

(e). (3 points) Use the graph to answer these questions.
As $x \to +\infty, r(x) \to$ _____; As $x \to -\infty, r(x) \to$ _____
As $x \to 3^+, r(x) \to$ _____; As $x \to 3^-, r(x) \to$ _____
As $x \to -2^+, r(x) \to$ _____; As $x \to -2^-, r(x) \to$ _____

(f). (1 point) What, if any, is the horizontal asymptote of $r(x)$? What does this tell you about p, q and/or how they relate? Be specific.

(g). (2 points) Write a possible equation for $r(x)$ taking into account your answers above.

Question 3 (4 points) For each question below, justify your answer. If your answer is "no," then explain why not and give a counterexample (visually or symbolically).

(a). (1 point) Suppose $\lim_{x \to 5^+} a(x) = 1 = a(5)$. Must $a(x)$ be continuous at $x = 5$.

(b). (1 point) Suppose $\lim_{x \to 5} b(x) = 1$. Must $b(x)$ be continuous at $x = 5$.

(c). (1 point) Suppose $\lim_{x \to 5^+} c(x) = \lim_{x \to 5^-} c(x)$. Must $c(x)$ be continuous at $x = 5$.

(d). (1 point) When is a function $d(x)$ continuous at $x = 5$? Explain conceptually **and** via limits.