Quiz 1 for Section 3

History of Mathematics

UCR Math-153-03, Spring 2019

1. Find the Egyptian fraction expansion for $\frac{7}{26}$.

   Note that there are infinitely many Egyptian fraction expansions of any fraction between 0 and 1. It turns out though that the greedy algorithm provides a nice expansion for $\frac{7}{26}$. Notice that $\frac{1}{4}$, which equals $\frac{7}{28}$, is strictly less than $\frac{7}{26}$, so let's write

$$\frac{7}{26} = \frac{1}{4} + d.$$

   Solving for d, we get that it must be $\frac{1}{52}$ which has a numerator of 1, so we're done.

2. The pentagonal numbers are given by $P(2) = 5$ and $P(n+1) = P(n) + 3n + 1$ for $n \geq 2$. Prove by induction that $P(n) = \frac{1}{2}(3n^2 - n)$.

First, our base case is covered since $P(2) = \frac{1}{2}(3(2)^2 - (2)) = 5$. Now suppose that for a *fixed* natural number $k \geq 2$ we have $P(k) = \frac{1}{2}(3k^2 - k)$. Then we have

$$\begin{aligned} P(k+1) &= P(k) + 3k + 1 \\ &= \frac{1}{2}(3k^2 - k) + 3k + 1 \\ &= \frac{1}{2}(3k^2 - k + 6k + 2) \\ &= \frac{1}{2}(3k^2 + 6k + 3 - k - 1) \\ &= \frac{1}{2}\left(3(k+1)^2 - (k+1)\right). \end{aligned}$$

So $P(n) = \frac{1}{2}(3n^2 - n)$ holds for $n = k + 1$ too, which by induction means it holds for all $n \geq 2$.

Quiz 1 for Section 2

History of Mathematics

UCR Math-153-03, Spring 2019

1. Find the Egyptian fraction expansion for $\frac{5}{26}$.

    Note that there are infinitely many Egyptian fraction expansions of any fraction between 0 and 1. It turns out though that the greedy algorithm provides a nice expansion for $\frac{5}{26}$. Notice that $\frac{1}{6}$, which equals $\frac{5}{30}$, is strictly less than $\frac{5}{26}$, so let's write

    $$\frac{5}{26} = \frac{1}{6} + d\,.$$

    Solving for d, we get that it must be $\frac{1}{39}$ which has a numerator of 1, so we're done.

There is another question on the back.

2. The hexagonal numbers are given by $H(2) = 6$ and $H(n+1) = H(n) + 4n + 1$ for $n \geq 2$. Prove by induction that $H(n) = 2n^2 - n$.

   First, our base case is covered since $H(2) = 2(2)^2 - (2) = 6$. Now suppose that for a *fixed* natural number $k \geq 2$ we have $H(k) = 2k^2 - k$. Then we have

$$\begin{aligned} H(k+1) &= H(k) + 4k + 1 \\ &= \left(2k^2 - k\right) + 4k + 1 \\ &= 2k^2 + 4k + 2 - k - 1 \\ &= 2(k+1)^2 - (k+1). \end{aligned}$$

   So $H(n) = 4n^2 - n$ holds for $n = k+1$ too, which by induction means it holds for all $n \geq 2$.