

Week Six Quiz

Linear Algebra
UCR Math-131, Fall 2020

1. Consider the ordered basis $\mathbf{u} = (3, i)$ and $\mathbf{v} = (1, 5)$ of \mathbf{C}^2 where $i^2 = -1$. Also consider the point $\mathbf{p} = (i, 2)$ expressed in terms of the standard basis of \mathbf{C}^2 , so $\mathbf{p} = i\mathbf{i} + 2\mathbf{j}$. What are the coordinates of \mathbf{p} in terms of the ordered basis $\{\mathbf{u}, \mathbf{v}\}$?

Let (a, b) be the coordinates of \mathbf{p} in terms of the ordered basis $\{\mathbf{u}, \mathbf{v}\}$. So we have $i\mathbf{i} + 2\mathbf{j} = a\mathbf{u} + b\mathbf{v}$, which means

$$\begin{aligned}(i, 2) \begin{pmatrix} (1, 0) \\ (0, 1) \end{pmatrix} &= (a, b) \begin{pmatrix} (3, i) \\ (1, 5) \end{pmatrix} \\ (i \ 2) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} &= (a \ b) \begin{pmatrix} 3 & i \\ 1 & 5 \end{pmatrix} \\ (i \ 2) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & i \\ 1 & 5 \end{pmatrix}^{-1} &= (a \ b) \\ (i \ 2) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{15-i} & \begin{pmatrix} 5 & -i \\ -1 & 3 \end{pmatrix} \end{pmatrix} &= (a \ b) \\ \begin{pmatrix} \frac{-2+5i}{15-i} & \frac{7}{15-i} \end{pmatrix} &= (a \ b) \\ \begin{pmatrix} \frac{-2+5i}{15-i} & \frac{7}{15-i} \end{pmatrix} &= (a, b) \\ \begin{pmatrix} \frac{-35+73i}{226} & \frac{105+7i}{226} \end{pmatrix} &= \mathbf{p} \end{aligned}$$