Week Three Quiz

Linear Algebra

UCR Math-131, Fall 2020

1. What does it mean for a set of vectors $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$ to *span* a vector space V?

   A set of vectors span a space if every vector in the space can be written as a linear combination as the vectors in that set. Spelled out explicitly, this means for every $\boldsymbol{v} \in V$ there exists some constants $a_1, \ldots, a_n$ in the base field such that $\boldsymbol{v} = a_1 \boldsymbol{v}_1 + \cdots + a_n \boldsymbol{v}_n$.

2. Do the vectors $\langle 1, 2, 3\rangle$, $\langle -1, 0, 1\rangle$, and $\langle 0, 0, 1\rangle$ span $\mathbf{R}^3$? Use techniques proven in class to answer this question, and please explicitly tell the grader what technique you're using.

   Let's show those vectors span $\mathbf{R}^3$ by writing an arbitrary vector $\langle a, b, c\rangle \in \mathbf{R}^3$ explicitly as a linear combination of those vectors. Let $\boldsymbol{i} = \langle 1, 0, 0\rangle$, $\boldsymbol{j} = \langle 0, 1, 0\rangle$, and $\boldsymbol{k} = \langle 0, 0, 1\rangle$, and let's name the vectors above as $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, and $\boldsymbol{v}_3$ respectively. Notice that $\boldsymbol{i} = \boldsymbol{v}_3 - \boldsymbol{v}_2$, $\boldsymbol{j} = \frac{1}{2}\boldsymbol{v}_1 + \frac{1}{2}\boldsymbol{v}_2 - 2\boldsymbol{v}_3$, and $\boldsymbol{k} = \boldsymbol{v}_3$, and so

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\begin{aligned}
\langle a, b, c\rangle &= a\boldsymbol{i} + b\boldsymbol{j} + c\boldsymbol{k} \\
&= a(\boldsymbol{v}_3 - \boldsymbol{v}_2) + b(\frac{1}{2}\boldsymbol{v}_1 + \frac{1}{2}\boldsymbol{v}_2 - 2\boldsymbol{v}_3) + c(\boldsymbol{v}_3) \\
&= (\frac{1}{2}b)\boldsymbol{v}_1 + (-a + \frac{1}{2}b)\boldsymbol{v}_2 + (a - 2b + c)\boldsymbol{v}_3
\end{aligned}
$$

   which is exactly what we sought to show.