NT Notes 4

3 1. Let p be a prime such that $p \geq 5$. Prove $24 \mid p^2 - 1$.
$(p-1)(p+1)$ and $2 \nmid p$ $3 \nmid p$ ...
↑ p

4 2. Show that for $n=3$ is the only number such that all of n, $n+10$, and $n+14$ are prime. (HINT: mod 3)

5* Take $a,b,c,d \in$ positive $\mathbb{N}$ such that $ab = cd$. Prove that $a+b+c+d$ is not prime.
Say $\frac{a}{c} = \frac{b}{d} = \frac{m}{n}$, so $a=mt$ $b=ms$ $c=nt$ $d=ns$, and
$mt + ms + nt + ns = (m+n)(s+t)$.

More straightforward stuff ish

*1 1. Prove that no prime can be written as the sum of two squares in two distinct ways. $p = a^2+b^2 = c^2+d^2 \Rightarrow p = (a-c)(a+c) = (b+d)(b-d)$ ...
Wait! This is hard?!

2. Find the remainder of $67!$ when divided by 71.

Lagrange's Thm
Fermat's Little
Wilson's
Thue $p = x^2 + 3y^2 \Rightarrow z^2 \equiv -3 \bmod p$

Wilson's theorem $(p-1)! \equiv_p -1$ $\quad 70 \cdot 69 \cdot 68 \cdot 67! \equiv -1$
$6 \cdot 67! \equiv -1$
$6^{-1} \equiv_p 12$

Note that my feedback on this homework is pretty limited since they were just computations. Not much exposition to give. But see my solutions for how to format computations nicely.