Nt Notes 7 Quadratic Reciprocity (section 5.6)

For distinct odd primes p, q, $\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\frac{1}{4}(p-1)(q-1)}$.

1. Evaluate (i) $\left(\frac{3}{53}\right)$ (ii) $\left(\frac{31}{641}\right)$ 641 prime

I got -1

2. Show that for an odd prime $p>3$ $\left(\frac{3}{p}\right) = \begin{cases} 1 & \text{if } p \equiv_{12} \pm 1 \\ -1 & \text{if } p \equiv_{12} \pm 5 \end{cases}$.

$$\left(\frac{3}{p}\right)\left(\frac{p}{3}\right) = (-1)^{\frac{1}{4}(p-1)(3-1)} = (-1)^{\frac{1}{2}(p-1)} \quad \text{and } p = 12k \pm 1, \pm 5 \ \}\text{ cases.}$$

$\uparrow$ $\left(\frac{1}{3}\right)=1$, $\left(\frac{2}{3}\right)=-1$ CASES! ⍨

3. Describe all primes p for which 5 is a quadratic residue. No, like in (2) find congruences to describe all p...

holds for $p=2$

$$\left(\frac{5}{p}\right)\left(\frac{p}{5}\right) = (-1)^{\frac{1}{4}(p-1)(5-1)} = (-1)^{p-1} = 1 \ \forall \text{ odd } p$$

So $\left(\frac{5}{p}\right)=1$ exactly when $\left(\frac{p}{5}\right)=1$, so when $p \equiv_5 \pm 1$. ✓

4* Prove that the congruence

$$(x^2-2)(x^2-17)(x^2-34) \equiv 0 \pmod{p}$$

has a solution for any prime p. 14

$$\left(\frac{34}{p}\right) = \left(\frac{2}{p}\right)\left(\frac{17}{p}\right) \checkmark$$

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4:14
4:31