Homework 2

Ordinary Differential Equations

UCR Math-046-E01, Summer 2018

1. What does it mean for a function $y(x)$ to be a *solution* to a differential equation? Demonstrate that $y = 2e^{-x} + xe^{-x}$ is a solution to the differential equation

$$y'' + 2y' + y = 0\,.$$

2. Can you tell me a solution to the differential equation $y' = ky$, where some k is some real number? (HINT: Don't try to *solve* the differential equation; that's excessive. All this is asking for is a function that is a constant multiple of its own derivative. I hope you recall an example of such a function from your previous calculus classes.) Do you think you can guess the *general solution* to this differential equation?

3. Explain why the differential equation $(y')^4 + y^2 = -1$ has *no* real solutions. By *explain*, I mean write down a few sentences that another human could easy read to become convinced that this differential equation has no solutions.

4. Draw the vector field corresponding to the differential equation

$$y' = y + x^2\,.$$

   This may be kinda tough, but just make your picture big and try your best. Note that this is similar to Example 2 in Paul's Online Notes on Direction Fields. For this differential equation though, the curves along which the derivative y' is constant will not be straight lines. What curves will they be?

5. Solve these two differential equations. (HINT: Notice that I'm not providing any initial conditions, so I'm asking for the *general* solution to each of these.)

$$y' - 6y = 4 \qquad\qquad xy' - 2y' = 2(y-4)$$

6. Recall what it means for a differential equation to be *homogeneous*. Demonstrate that the following differential equation is *not* homogeneous.

$$y' = \frac{x^2 - yx + 9}{y^2}$$

7. Solve these three differential equations. (HINT: Notice that I'm providing you with initial conditions for these differential equations, so these are examples of *initial value problems*, and I'm asking for a *particular* solution to each of these. Don't forget to specify the domain on which each of your solutions is defined.)

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{y+t}{t} \quad \text{where} \quad y(e) = 3e$$

$$\mathrm{e}^t - yy' = 0 \quad \text{where} \quad y(0) = 3$$

$$xyy' = x^2 + y^2 \quad \text{where} \quad y(1) = -2$$

You may want more practice solving these particular types of differential equations. If you do, you can find more practice problems by looking for separable and homogeneous differential equations on WeBWorK, or in any textbook on ordinary differential equations, or by searching on the internet.