

Final Exam

Precalculus: Functions, Geometry, Trigonometry, & Modelling
UCR Math-005-E01, Summer 2020

There are six mathematics prompts on this exam, one prompt per page, and one special prompt ★ at the end asking about what grade I should give to UCR for you. You'll upload photos of your responses to the six math prompts to Gradescope, but you'll email me about ★.

gradescope.com/courses/125651

There are a couple purposes of this exam. First its purpose is to aid you in your own reflection on your mastery of the course topics.

Another purpose is to provide me a document to read so that I may assess your mastery of the course topics. So, a practical point, write your responses so that I can read them. I'm not a robot, looking only for right answers in boxes. I'm just trying to figure out what you know. Give me all the evidence that you can that you've learned the material of this course. I WANT YOU TO PASS THIS COURSE. Imagine that I'm a lawyer who has to argue to the University for the best grade I can give you with your exam in my hand. Talk to me in your responses on this exam. Even if you are unsure how to perfectly answer a question, tell me what you're thinking about, and tell me what you *do* know that could help answer the question.

And try to relax. If you start to feel stressed, lean back in your chair, close your eyes and take ten deep breathes; center yourself. It's just a math test, and you're doing great. :)

1. Answer the following questions.

(a) How many roots does the following function f have?

$$f(x) = \begin{cases} 4 - x^2 & \text{for } x < 1 \\ e^x & \text{for } x \geq 1 \end{cases}$$

(b) If n is a thirty-seven digit number, what is the greatest integer less than $\log_{10}(100n^2)$?

(c) If $\cos(\theta) = \frac{3}{7}$, what two values could $\tan(\theta)$ be?

(d) If a painter can paint 12 ft^2 in 5 minutes, at what rate does this painter paint in terms of yards^2 per hour?

(e) If the Earth is rotating at about 1000 mph and you are standing at a latitude of 34° N , how fast are you moving relative to the center of the earth?

(f) What's an equation for a line that passes through the point $(3, 4)$ and makes an angle of $\frac{\pi}{3}$ radians with the x -axis?

(g) If a population of bacteria currently has 123 456 789 members, but doubles every hour, how many bacterial will there be in 3.141592 hours?

(h) What angle is $\arcsin(\sin(17\pi/4))$ equal to?

2. A standard gold bar, according to the US Mint, measures $7 \times 3\frac{5}{8} \times 1\frac{3}{4}$ inches and weighs 400 oz. The market value of gold as of this weekend is \$1940.40 per ounce.

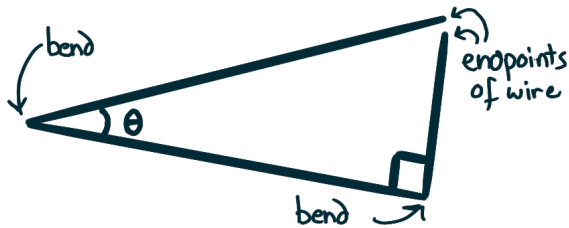
- (a) How much money is a standard bar of gold worth?
- (b) The minimum wage in California is currently \$12 per hour. There are about 52 weeks in a year. If you work eight hours a day for five days a week, saving all the money you make, in how many years will you be able to purchase a bar of gold?
- (c) Suppose you want to buy a scale replica of a standard bar of gold to wear around your neck on a chain, and that you have a spare \$2300 to spend on this. What are the dimensions of the scaled-down bar of gold that you could buy?



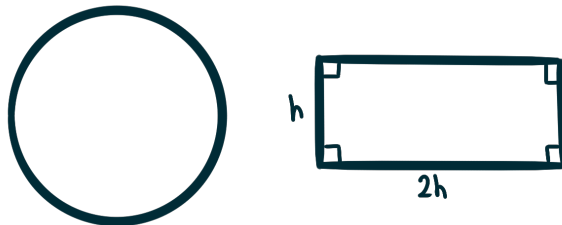
3. Let $(0, 0)$ be the coordinates of a port in Ventura CA, and let the vectors $\langle 1, 0 \rangle$ and $\langle 0, 1 \rangle$ point in the directions of east and north respectively. Suppose a boat leaves the port in Ventura headed towards Cavern Point on Santa Cruz Island with a heading of $\langle -1, -1 \rangle$ and a speed of 40 mph.
- (a) Using the boat's speed and heading, write an equation for the position of the boat at time t hours after departure.
 - (b) If we know the boat reaches Cavern Point after half an hour, what must the coordinates of Cavern Point be in this Cartesian coordinate system?
 - (c) Impose a polar coordinate system. What are the coordinates of Cavern Point with this polar coordinate system?
 - (d) Anacapa Island Lighthouse is at the coordinates $(-9, -6)$. What direction (vector) would the boat have to travel in to reach this lighthouse from Cavern Point?
 - (e) Along its original trip from Ventura to Cavern Point, what is the closest the boat gets to Anacapa Island Lighthouse?

4. Anita has a 150m piece of wire.

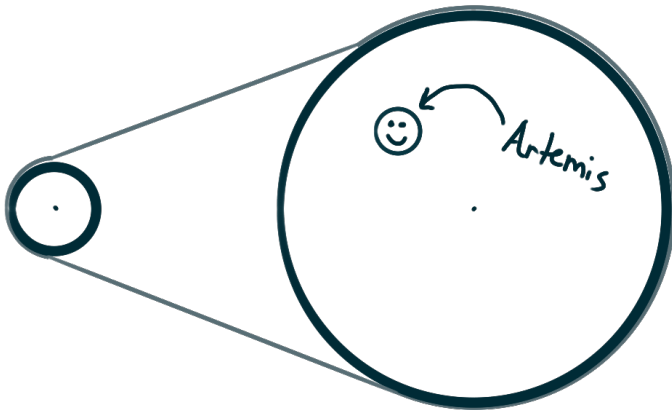
- (a) Anita bends the wire at an angle θ such that $\tan(\theta) = \frac{5}{12}$, and then bends the wire once more to create a right triangle as illustrated. What must the side-lengths of this triangle be?



- (b) Now Anita wants to cut the wire into two pieces, bending one piece into a circle, and bending the other piece into a rectangle that is twice as wide as it is high. Where should she cut the wire so that the total amount of area enclosed by these two shapes is minimal? What's the circumference of the resulting circle?



5. Suppose you have the system of rotating wheels connected by violet belts illustrated below. The large wheel has a radius of 23 meters and the small wheel a radius of 1 meter. There's a fella named Artemis sitting on the big wheel about 11 meters from the center. The large wheel is being powered and is rotating at 23 revolutions per minute.



- How fast is the outer edge of the large wheel moving?
- How fast is the outer edge of the small wheel moving?
- How fast is Artemis moving?
- How fast is the small wheel rotating? I.e. How many rotations is it making per minute?
- How many times has Artemis gone around the large wheel after one minute?

6. Consider the following function f .

$$f(x) = 3 \cos\left(\frac{\pi}{3}(x-5)\right) + 2$$

- (a) Write f as a composite of the following functions for whatever choices of a and b you need.

$$\cos(x) \quad t_a(x) = x + a \quad s_b(x) = bx$$

- (b) What is the domain of f ? What is its range? Recalling that cosine has a period of 2π , What is its period of f ?
- (c) What is the amplitude of f ?
- (d) If you haven't already, without using software, graph f .
- (e) Can you tell me why f does not have an inverse with a range of all real numbers?
- (f) Write down a formula for an inverse function f^{-1} of f with domain equal to the entire range of f . What is a possible choice of range of your f^{-1} ?

- ★ For this class, what grade should I submit to UCR for you? Don't respond to this question here, but instead answer this in an email to me after you upload your final exam responses to Gradescope.

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Note that your answer to this should reflect your mastery of skills and understanding of the concepts *now*, and should have nothing to do with past performance in this course. For example it doesn't matter if you did poorly on one of the quizzes, so long as you studied and learned what you were being quizzed on by now.

To develop a common language, and remembering that a purpose of this class is to prepare you to learn calculus, here's how I would describe each of the letter grades in terms of a personal reflection.

A I've done very well and am ready to take calculus.

B I've done well, but there are notably a couple skills I still haven't mastered or concepts I still haven't internalized, but still ready to take a calculus class.

C There are a few skills I haven't mastered and things I still don't understand, but I should be okay taking a calculus class.

F I'm concerned that my lack of fluency with the topics of this class will hinder my understanding of calculus.

Tell me *why* you chose the grade that you did. In particular, if you did not assign yourself an **A**, tell me what specific topics you don't think you've mastered/understood well enough to merit that **A**.