

Last Name, First Name

Discussion Section

Student ID

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### Worksheet 4 • Perpendicular Lines and the Horizon

1. Draw in the coordinate plane the line that intersects the points  $(1, 2)$  and  $(5, 8)$ . Find an equation for this line.
2. Find an equation for the line perpendicular to this line that intersects the point  $(2, 7)$ .
3. Draw a line in the plane. Call this line  $L$ . Draw a point,  $p$ , that is not on  $L$ . Let  $L_{\perp}$  be the line perpendicular to  $L$  that intersects  $p$ . The line  $L_{\perp}$  will intersect  $L$  at a point  $q$ . Use basic geometry to argue that  $q$  is the point on  $L$  that is closest to  $p$ .
4. Use the previous problem to find the point on the line given by

$$y = 2x + 1$$

that is closest to the point  $(1, 4)$ .

5. Suppose that  $p$  and  $q$  are two points on a circle of radius  $r$  with center  $O$  and that  $p$ ,  $q$ , and  $O$  are not co-linear. Let  $m$  be the midpoint of  $\overline{pq}$ , the line segment with endpoints  $p$  and  $q$ . Show that  $\overline{Om}$  and  $\overline{pq}$  are perpendicular.
6. Suppose that a line  $L$  intersects a circle  $C$  centered at  $O$  in exactly one point,  $p$ . Use Problem 3 to show that  $\overline{Op}$  is perpendicular to  $L$ .
7. Recall that the earth's radius is 4,000 miles. Suppose that you stand  $h$  feet above sea level and you look out across the ocean. What is the length of your line of sight? Make sure that you give the answer in miles. How far is the horizon away when you stand 6 feet above sea level? How about 9 feet? How about 100 feet?
8. In the previous problem, You calculated the distance to the horizon. This is not the distance one would have to travel on the earth to get to the horizon. Can you estimate this distance? This is to say, at most how far off is the length of the line of sight from length of the shortest arc that starts at your position and goes to where you see the horizon? Hint: You'll need to draw a picture and think geometrically, remembering a critical fact about triangles and arcs of circles.