Last Name, First Name	Discussion Section	Student ID

Worksheet 8 ● Estimating Values of Trigonometric Functions

1. A point p on the unit circle is said to be a principle dyadic angle if the radian measure of p is zero or $\frac{2\pi}{2^n}$ for some natural number n. Draw a unit circle. Then mark and label the first seven non-zero principle dyadic angles on it, giving both degree and radian measures of the labeled angles.

2. Use the half-angle formula for the cosine function to calculate $\cos(90°)$, $\cos(45°)$, $\cos(22.5°)$, $\cos(11.25°)$, and $\cos(5.625°)$.

3. Mark off all angles in the first quadrant on a circle that are multiples of $5.625°$.

4. A dyadic angle is an angle that is a sum of principle dyadic angles, this is to say that its measure in radians is zero or $2\pi \times \frac{m}{2^n}$ for some natural numbers m and n. Show that the sum and average of two dyadic angles is again a dyadic angle.

5. By averaging dyadic angles, one can quickly approximate any angle by dyadic angles using a method very similar to the bisection method. This is to say that given an angle θ, one can find dyadic angles A and B such that θ lies between A and B and A and B are as close to each other as we like. Approximate $18°$ by averaging dyadic angles, then write the approximating dyadic angles as sums of principle dyadic angles.

6. Approximate $\cos(18°)$ and $\sin(18°)$. At worst, how far off from the exact answer can your approximation be? Neglect the error that you get in your calculation of the trigonometric functions on the approximating dyadic angles. Test the accuracy of your approximation with a calculator.