

Last Name, First Name

Discussion Section

Student ID

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### Worksheet 9 • Restriction of Domain

**1.** Graph the function  $f(x) = (x + 1)^2$ . Then reflect the graph of  $f$  about the line  $y = x$ . (Comment: When you reflect the graph about the line  $y = x$ , you interchange the roles of the  $x$  and  $y$  coordinates.) Is the reflection of this graph the graph of a function?

**2.** Restrict the domain of  $f$  given by  $f(x) = (x + 1)^2$  in the simplest way possible so that  $f$  becomes invertible on that domain and the range of the inverse is an interval containing zero.

**3.** Consider the function,  $f$ , that projects the unit circle,  $\mathcal{C}$ , onto the  $x$ -axis. This is the function

$$f: \mathcal{C} \rightarrow [-1, 1] \quad \text{defined by} \quad f((a, b)) = a.$$

Is this function invertible? If not, find the largest arc,  $\mathcal{A}$ , on the circle and containing  $(1, 0)$  where it is invertible. You should highlight this arc  $\mathcal{A}$ .

**4.** Consider the function,  $g$ , that projects the unit circle onto the  $y$ -axis. This is the function

$$g: \mathcal{C} \rightarrow [-1, 1] \quad \text{defined by} \quad g((a, b)) = b.$$

Is this function invertible? If not, find the largest arc,  $\mathcal{B}$ , on the circle and containing  $(0, 1)$  where it is invertible. You should highlight this arc  $\mathcal{B}$ .

**5.** Denote respectively by  $f^{-1}$  and  $g^{-1}$  the inverse of the restriction of  $f$  on  $\mathcal{A}$  and the inverse of the restriction of  $g$  on  $\mathcal{B}$ . Calculate

$$f^{-1}\left(\frac{1}{3}\right), \quad f^{-1}\left(-\frac{1}{3}\right), \quad g^{-1}\left(\frac{2}{3}\right), \quad \text{and} \quad g^{-1}\left(-\frac{2}{3}\right).$$

**6.** Calculate

(a)  $f^{-1}\left(f\left(\left(\frac{2}{5}, \sqrt{21}/5\right)\right)\right)$

(b)  $f^{-1}\left(f\left(\left(\frac{2}{5}, -\sqrt{21}/5\right)\right)\right)$

(c)  $g^{-1}\left(g\left(\left(\frac{2}{5}, \sqrt{21}/5\right)\right)\right)$

(d)  $g^{-1}\left(g\left(\left(-\frac{2}{5}, \sqrt{21}/5\right)\right)\right)$ .

(e)  $g^{-1}\left(f\left(\left(\frac{2}{5}, \sqrt{21}/5\right)\right)\right)$

(f)  $g^{-1}\left(f\left(\left(\frac{2}{5}, -\sqrt{21}/5\right)\right)\right)$

(g)  $f^{-1}\left(g\left(\left(\frac{2}{5}, \sqrt{21}/5\right)\right)\right)$

(h)  $f^{-1}\left(g\left(\left(-\frac{2}{5}, \sqrt{21}/5\right)\right)\right)$ .