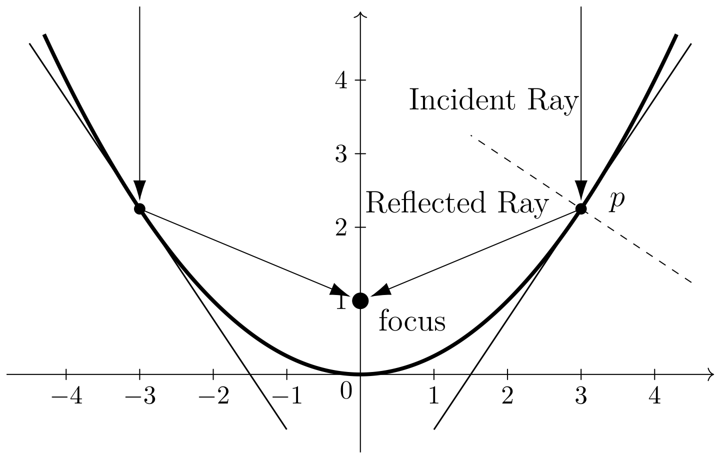
Last Name, First Name	Discussion Section	Student ID

Worksheet 11 ● Tangency and Telescopes

Parabolic mirrors are important in telescope construction because all incoming parallel light rays to a reflective parabolic surface reflect off the surface onto the same point, the focus of the parabola. This enables light from a source to be gathered over a large region. In this worksheet we will seek to understand this reflection principle. We need one fact about reflections in two dimensions that we can regard as experimentally determined: When light falls onto a reflective curve, hitting a point p, it reflects in such a way that the incident ray and the reflected ray are reflections across the line intersecting p and perpendicular to the line tangent to the curve at p. The picture below illustrates this principle.



1. Graph the function f given by $f(x) = 3x^2$ and draw at least three incoming (incident) light rays parallel to the axis of symmetry of the graph of f, the line $x = 0$.

2. Where should the incident light ray that moves along the line $x = 0$ reflect?

3. Find the line, L, tangent to the graph of f at the point $(2, 12)$.

4. Find an equation for the line, $L_\perp$, perpendicular to L that intersects $(2, 12)$.

5. Find an equation for the path of motion of the reflection across $L_\perp$ of the incident light ray parallel to the y-axis and intersecting $(2, 12)$.

6. Redo your above calculations, where L is now tangent to the graph of f at the point $(a, f(a))$, $L_\perp$ is perpendicular to L and intersects $(a, f(a))$, and the incident light ray parallel to the y-axis intersects $(a, f(a))$. Show that all reflected light rays intersect the same point, the focus.

7. Let A be a positive real number. Redo the previous problem, but now where f is given by $f(x) = Ax^2$.