The weight of paint is proportional to its volume. Paint is always applied at the same thickness*. It takes 10 ounces of paint to paint a ball that has a volume of 3 cubic feet. How many ounces of paint does it take to paint a ball that has a volume of 24 cubic feet?

When the problem says "the weight of paint is proportional to its volume," this really means that, since we'll be dealing with rations, we may use weight and volume interchangably. In more detail, for any glob of paint with volume V and weight W, the ration V/W is a constant, so $V = kW$ for some number k. The idea is that anytime we would need to use W in a ration, we could just use $\frac{1}{k}V$ insted, and the factors of $1/k$ would cancel out.

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* This should probably also say "and the thickness is negligably small." Otherwise, the thickness is some constant and this problem is more complicated.

Now lets do a quick aside concerning units across dimensions.

Suppose you have some 3-dimensional geometric object, like a ball or cube or coffee mug or whatever. Now suppose you scale the object by a factor of n, so you multiply all the 1-dimensional measurements of the object (length, height, radius, etc) by n. What happens to the values of 2-dimensional measurements like surface area, or cross-sectional area? What happens to 3-dimensional measurments like volume? It turns out they increase by a factor of n^2 and n^3 respectively. So if the surface area of the original ~~ball~~ object was 7, the surface area of the scaled object is $7n^2$. If the volume of the original object was 5, the volume of the scaled object is $5n^3$. You can see an example of this by starting with a cube of side-length 1 (so the surface area is 6 and volume is 1) and compare these to the surface area and volume of a cube of side-length 2.

Back to the problem at hand, the volume of the ball is a 3-dimensional measurement, while the amount of paint we are applying to the ball will be a 2-dimensional measurement since we want to think of it as the surface area of the ball.*

If the big ball is n times bigger than the smaller ball (1-dimensional scaling), then big ball's surface area is n^2 times bigger, and the big ball's volume is n^3 times bigger. But we know how much big the volume is. The volume is scaled by a factor of $24/3$, which is 8. So $n^3 = 8$. This means $n=2$, so really the big ball has twice the radius of the small ball. But we need to scale the surface area, which is proportional to that 10 ounces of paint. If it takes 10 ounces to paint the smaller ball, it must take $10 \cdot n^2$ ounces, which is 40 ounces, to paint the larger ball.

* The controversy here is that if the paint has "constant thickness" as the problem statement suggests, then this should really be a 3-dimensional measurement.