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2.3.4 Suppose that f is a periodic function with principal period equal to five. What is the principal period of $f \circ S_7$?

By definition, the principal period of $f \circ S_7$ is the smallest positive number p such that for all x in the domain of $f \circ S_7$,

$$f \circ S_7(x) = f \circ S_7(x+p).$$

So wee need to find p such that $f \circ S_7(x) = \ldots$

$\ldots = f(7x) = f(7(x+p)) = f(7x + 7p)$. But f has principal period five, so

$$f(\text{anything}) = f(\text{anything} + 5).$$

In particular, $7x$ can be our "anything", so $7p = 5$, and our principal period p of $f \circ S_7$ must be $5/7$.

2.4.4 Write the function $f(x) = |2x+1| - |x-3|$ as a piecewise-defined function, and use this to find all x such that $|2x+1| > |x-3|$.

Recall that the absolute value function is defined like

$$|\text{stuff}| = \begin{cases} \text{stuff} & \text{if } \text{stuff} > 0 \\ -\text{stuff} & \text{if } \text{stuff} \leq 0 \end{cases}.$$

Let's use that to write f by "splitting" the $|2x+1|$:

$$f(x) = |2x+1| - |x-3| = \begin{cases} 2x+1-|x-3| & \text{if } 2x+1 > 0 \\ -(2x+1) - |x-3| & \text{if } 2x+1 \leq 0 \end{cases}$$

$$\Longrightarrow \quad f(x) = \begin{cases} 2x+1-|x-3| & \text{if } x > -\frac{1}{2} \\ -2x-1-|x-3| & \text{if } x \leq -\frac{1}{2} \end{cases}$$

Now let's use the definition again to rewrite each "piece" of $f(x)$ above by splitting the $|x-3|$:

$$f(x)=\begin{cases} 2x+1-|x-3| & \text{if } x>-\frac{1}{2} \\ -2x-1-|x-3| & \text{if } x\le -\frac{1}{2} \end{cases}$$

$$\Rightarrow f(x)=\begin{cases} 2x+1-(x-3) & \text{if } x>-\frac{1}{2} \text{ AND } x-3>0 \\ 2x+1-(-(x-3)) & \text{if } x>-\frac{1}{2} \text{ AND } x-3\le 0 \\ -2x-1-(x-3) & \text{if } x\le-\frac{1}{2} \text{ AND } x-3>0 \\ -2x-1-(-(x-3)) & \text{if } x\le-\frac{1}{2} \text{ AND } x-3\le 0 \end{cases}$$

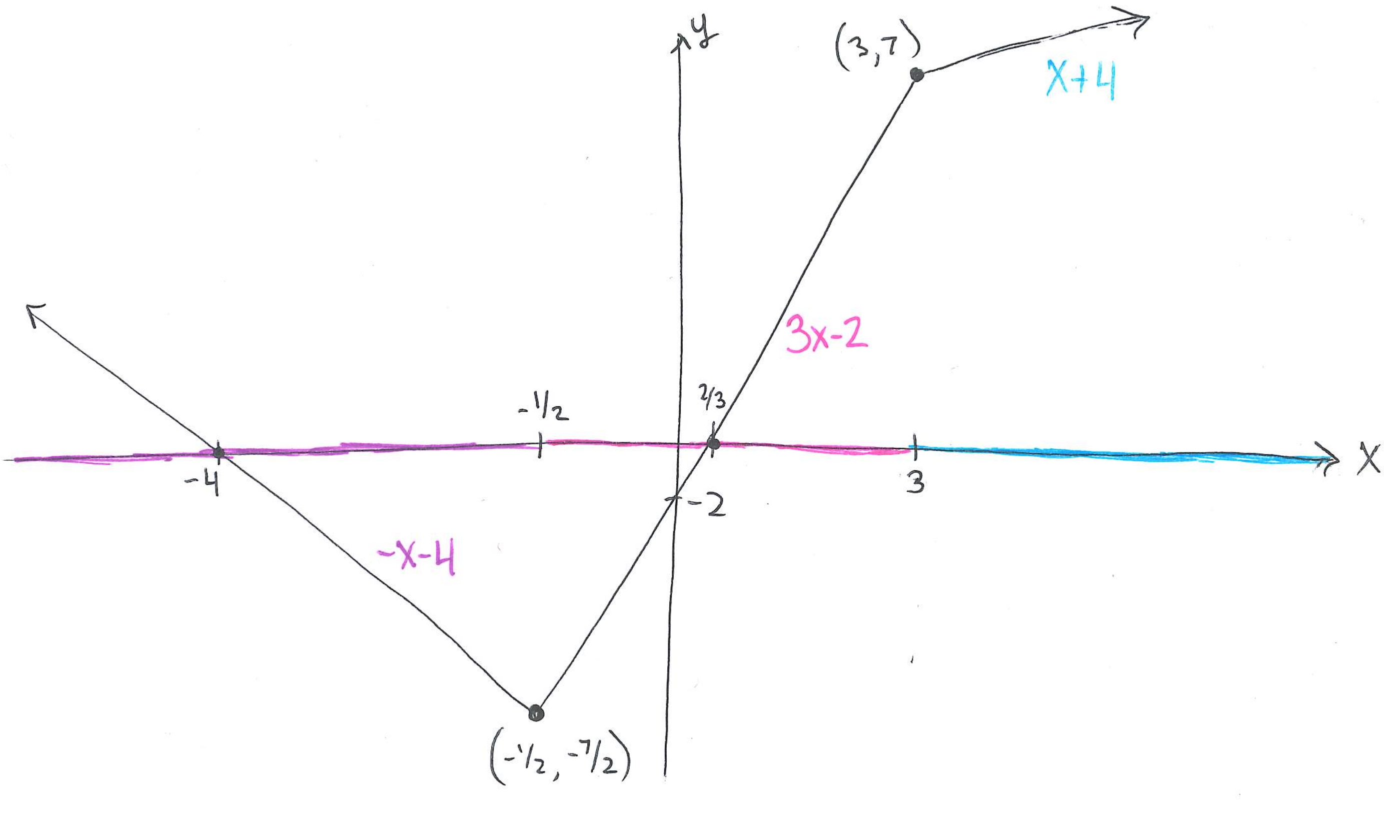
$$\Rightarrow f(x)=\begin{cases} x+4 & \text{if } x>-\frac{1}{2} \text{ AND } x>3 \\ 3x-2 & \text{if } x>-\frac{1}{2} \text{ AND } x\le 3 \\ -3x+2 & \text{if } x\le-\frac{1}{2} \text{ AND } x>3 \quad (*) \\ -x-4 & \text{if } x\le-\frac{1}{2} \text{ AND } x\le 3 \end{cases}$$

Now we can clean up the conditions of the function f, and ~~that~~ notice that the condition $(*)$ never happens, so

$$f(x)=\begin{cases} x+4 & \text{if } \underline{x>3} \\ 3x-2 & \text{if } \underline{-\frac{1}{2}<x\le 3} \\ -x-4 & \text{if } \underline{x\le -\frac{1}{2}} \end{cases} .$$

Now that we've written f as a piecewise function, we need to find all x such that $|2x+1| > |x-3|$. Notice that by our original presentation of f, this is the same as asking to find all x such that $f(x) > 0$.

Accurately graphing f will help us find such x.



The ~~line~~ piece $-x-4$ crosses the axis at $x=-4$, and the piece $3x-2$ crosses the x-axis at $x=2/3$, so it's clear from the graph of $f(x)$ that $|2x+1| > |x-3|$, that $f(x) > 0$, for all x in $(-\infty,-4) \cup (2/3, \infty)$.