Mike Pierce

Homework 1

1.4.6 Can a value of a be chosen so that

$$\begin{cases} x + 2y = 9 \\ 3x + 6y = a \end{cases}$$

has exactly one solution?

No, no such value of a exists. Each of these equations represent a line in the plane, and the solutions to this system give us the points in the plane when these lines intersect. Notice that if we rewrite the second equation by dividing through by 3, we get the system

$$\begin{cases} x + 2y = 9 \\ x + 2y = a/3 \end{cases}.$$

You might recognize immediately that these lines are parallel. If not, you could rewrite the equations in slope intercept form,

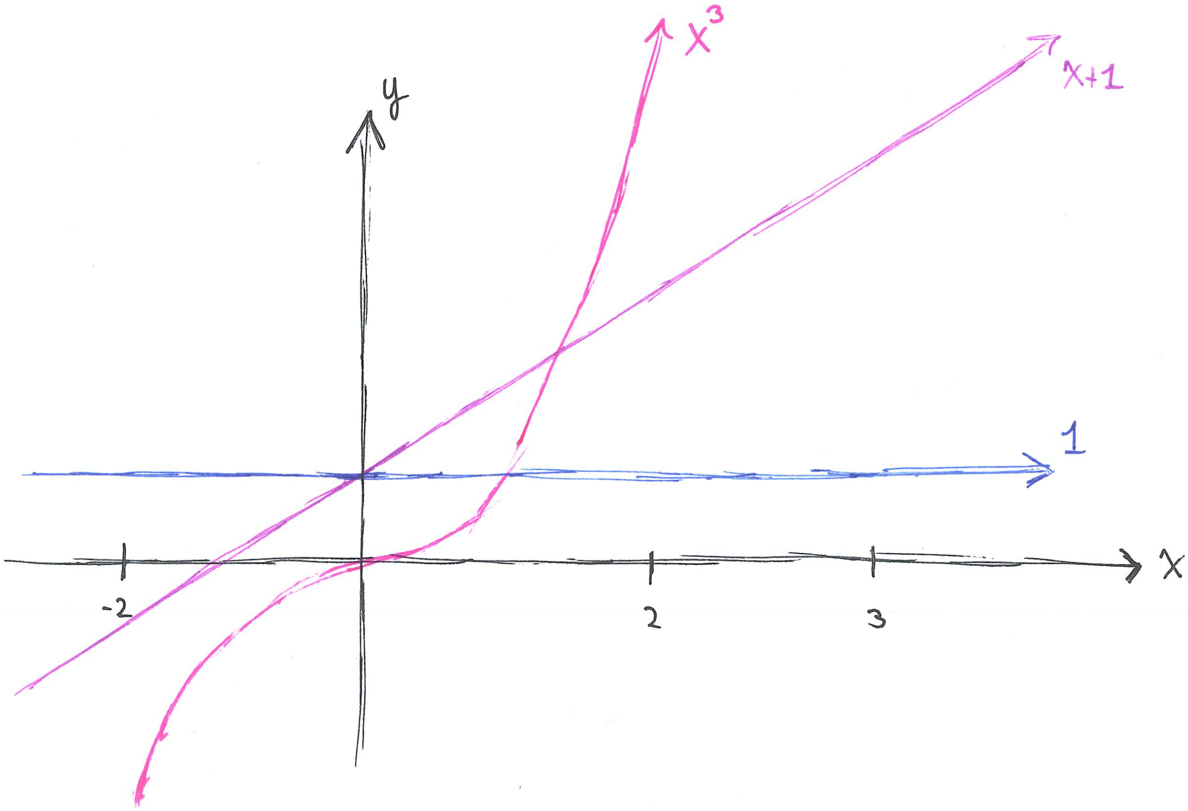
$$\begin{cases} y = -\frac{1}{2}x + \frac{9}{2} \\ y = -\frac{1}{2}x + \frac{9}{6} \end{cases}$$

and see that they're parallel because they have the same slope. Since the lines are parallel, they either don't intersect (so the system has no solutions), or the lines lie directly on top of each other (and so the system has infinitely many solutions). Either way, we can't pick a such that the system has exactly one solution.

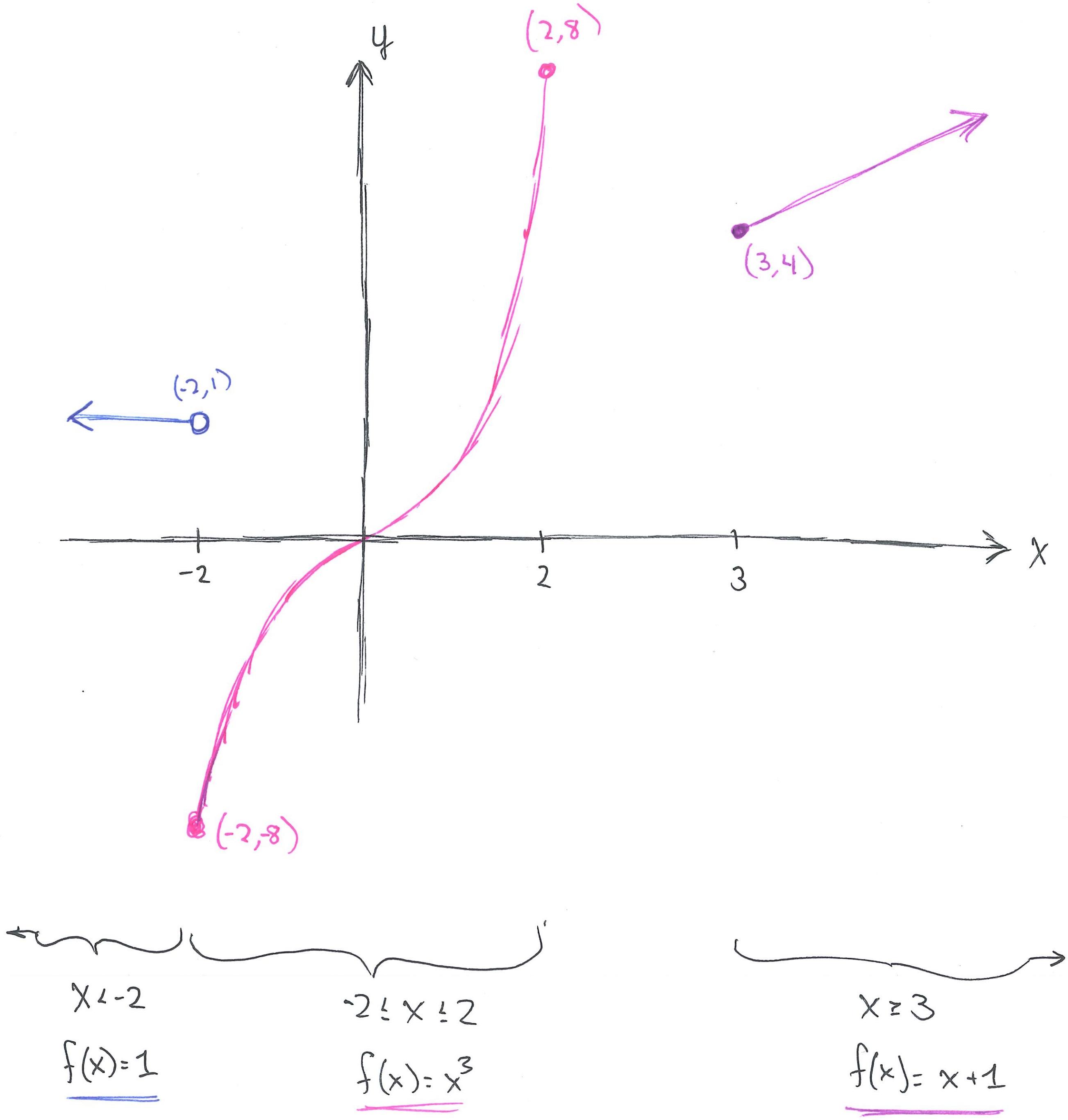
1.5.3 Sketch the graph of the function f given by

$$f(x) = \begin{cases} 1 & \text{if } x < -2 \\ x^3 & \text{if } -2 \le x \le 2 \\ x+1 & \text{if } x \ge 3 \end{cases}$$

First, let's just graph the individual ~~curve~~ pieces of $f(x)$ without considering the domain restrictions.



Now to think about the parts of the domain of f for which these pieces are defined.



Note that the interval (2,3) is not part of the domain of f.