Homework Four

Precalculus: Functions, Geometry, Trigonometry, & Modelling
UCR Math-005-E01, Summer 2020

Please upload your responses to these prompts to Gradescope before 8am next Monday morning. That way I have time to read over them to prepare for the coming week. You will upload all three of these sections under a single submission on Gradescope. When uploading the three sections, please keep them in the order listed here as Reflection, then Writing, then Exercises. There are (terse) solutions to some exercises at the end of this document.

gradescope.com/courses/125651

Reflection

First, do you think you've understood the math for this homework? Please respond to these in detail.

Is there anything you'd like me to look at more carefully in your Exercise and Writing responses? If there is, please indicate where it is in the rest of your submission. Like, circle it with a bright color or something.

Now since we're more than half-way through the term, let's do a more thorough check-in. At the end of the term, since I have to report something to UCR, I'll ask *you* what letter grade you deserve for this class. But reflecting on your overall progress like this, *grading yourself,* is a non-trivial skill, so you should start thinking about this now. Please reflect on your work in the course, on how much you've learned, on

the skills you've mastered and on the concepts you've better understood, but also on the skills you haven't mastered and concepts you still don't understand. How do *you* think you're doing in this class?

Just to develop a common language in case you need to think in terms of letter grades, and remembering that the purpose of this class is to prepare you to learn calculus, here's how I would describe each of the letter grades:

A I'm doing just fine :)

B I'm doing well, but there are a couple skills I still haven't mastered or concepts I still haven't internalized.

C There are a few skills I haven't mastered and things I still don't understand, but I should still be okay going into a calculus class.

F I'm concerned that my lack of fluency with the topics of this class will hinder my understanding of calculus.

Writing

1. (MODELLING & RESEARCH) The COVID-19 pandemic has been described as *growing exponentially*. For some reading on this, see

    Why outbreaks like coronavirus spread exponentially, and how to "flatten the curve" by the Washington Post

    The Coronavirus Pandemic Is Not Exponential via AIER

    But what exactly does this mean? *What* is growing exponentially? Death toll? Death rate? Infection total? Infection rate? Infection rate per capita? What is the relationship between these data? (that

one is secretly a calculus question) Are any of these truly exponential (since you've now learned the meaning of this word), or would the data better modelled differently? If so, how?

I'm asking you many questions here but I don't want you to feel compelled to answer all of them or limited to answering only them. Really, generally, I want you to provide me a numerical analysis of the current pandemic using some of the mathematical fluency you've gained in this class. And for answering this prompt you'll certainly want to find and cite other resources than those two I linked to. (are those resources even good?)

2. (PEDAGOGY) What is a logarithm? I think if you ask this to the typical person they couldn't provide an answer. But you are no longer the typical person. Suppose your neighbor's cousin's godchild is just now learning about logarithms in high school, but is confused about *what they are*. Can you explain here, as if explaining to this high school student, what a logarithm is?

Exercises

1. (UW) From the University of Washington's Math120 book, page 142 (pdf page 162), work through the exercises:

<div align="center">

10.3   10.5   10.6

</div>

2. (UW) From the University of Washington's Math120 book, page 152 (pdf page 172), work through the exercises:

<div align="center">

11.1   11.2   11.3

</div>

3. (CALCULATIONS) Without using a calculator, answer the following questions. Note that not every calculation will result in an integer: some might still need to be written in terms of logs or exponentials, but using the arithmetic rules of logs and exponentials there may certainly be a cleanest way to write them, and it's an exercise in your fluency with this arithmetic that you intuit when there is a cleaner way.

(a) What is the value of $\log_2(64)$?

(b) What integer is $\log_7(50)$ closest to?

(c) What integer is $\log_2(\pi^2)$ closest to?

(d) What is the greatest integer less than $\log_{10}(x)$, where x is some 23-digit number? (HINT: it doesn't matter which 23 digit number x actually is. Why is this?)

(e) What must x be if $\log_3(5) = \log_2(x)$?

(f) What must z be if $3^{(5^z)} = 7$?

(g) What must y be if $e^y = 10^5$?

(h) What is the value of $\log_4(100) - 2\log_4(5)$?

(i) What must ζ be if $\log_6(\zeta + 5) + \log_6(\zeta) = 2$? (Take care that you recall the domain of $\log_6$)

(j) (KINDA TOUGH) What must ℓ be if $2^\ell - 2^{-\ell} = 1$?

4. (UW) From the University of Washington's Math120 book, page 163 (pdf page 183), work through the exercises:

> 12.2   12.3   12.5   12.6   12.10   12.11

5. (RECREATIONAL) Three friends Anita, Becca, and Charleston are challenged to a game by the Game Maestro. The Game Maestro places two colored dots on each of the friends' foreheads and tells the friends that each dot is either blue or yellow, but neither color is used more than four times. He then places the three friends in a circle so that each of them can see the dots on their friends' foreheads, but not on their own. Then the game goes like this: The Maestro will ask the friends in turn, first Anita, then Becca, then Charleston, then Anita again, then Becca again, and so on, if they know the colors of the dots on their foreheads. If someone responds "no," the Maestro asks the next person. If someone responds "yes" and is right, the friends win! Whereas if someone responds "yes" and is wrong, all three friends will be banished to the shadow realm.

   The friends were given no time to strategize, but they begin playing. Their responses in turn are

> no   no   no   no   yes

   and the three friends win! Whare are the colors of the dots on Becca's forehead?

Some Solutions

Note that there are solutions to the University of Washington's book's exercises on page 291.

3. (a) 6

   (b) 2

   (c) 3

   (d) 22

   (e) $2^{\log_3(5)}$ I thought there was a cleaner way to write this one, but I couldn't figure it out before I got bored. Like, it seems odd to have a log in the exponent.

   (f) $\log_5(\log_3(7))$

   (g) $5\ln(10)$

   (h) 1

   (i) Using arithmetic ζ must be a solution to the polynomial $\zeta^2 + 5\zeta = 36$, but only one of them because the domain of log consists only of positive numbers. So $\zeta = 4$.

   (j) $\ell = \log_2\left(\frac{1}{2}(1 + \sqrt{5})\right)$

5. Feel free to talk to Mike about this one ;)